Sources of energetic particles in fusion Plasmas

Lars-Göran Eriksson
Department of Space, Earth and Environment
Chalmers University of Technology
Gothenburg, Sweden

Sources of energetic ions

- Nuclear reactions, especially alpha particles generated in D-T reactions
  - Good confinement of alpha particles is obviously essential for a fusion plasma
  - Fast ion instabilities driven by is a concern for ITER alpha particle heating
- Neutral Beam Injection (NBI) for auxiliary plasma heating
  - The work horse for plasma heating in most present day devices
  - Mostly ion heating in current day devices E_{inj} ~ 100 keV;
  - Less ion heating in ITER E_{inj} ~ 1MeV, especially in ramp-up phase
  - Can induce strong plasma rotation and non-inductive currents
- Acceleration of ions by Radio Frequency (RF) for auxiliary heating
  - Complicated wave physics + wave particle interaction, relies on velocity space diffusion for absorption; antenna needs to be near the plasma
  - Frequently creates ions with energies in the MeV range mainly perpendicular to $\vec{B}$
  - Often predominant electron heating trough collisions with accelerated ions, but can provide ion heating in the right circumstances
  - Some potential for current profile control

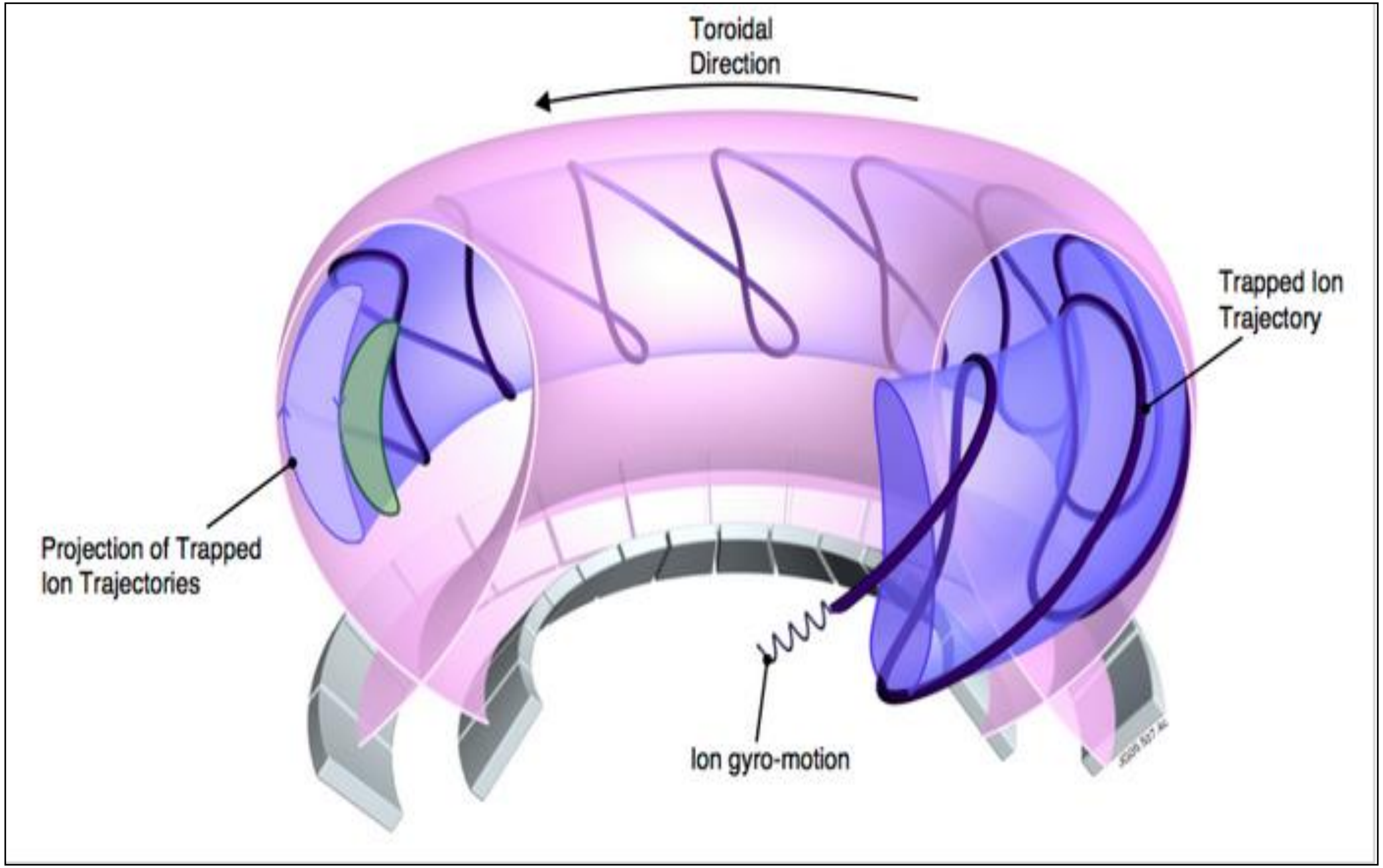
OK that's it!

Outline

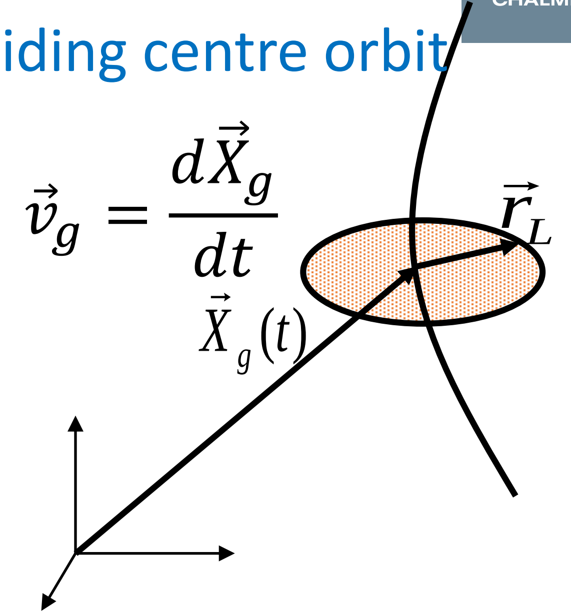
- Fast ion orbits
- The orbit averaged Fokker-Planck equation
- Alpha particles and classical slowing down
- Neutral Beam injection, the basics
- Ion Cyclotron Resonance Frequency (ICRF) Heating, wave propagation and wave-particle interaction.

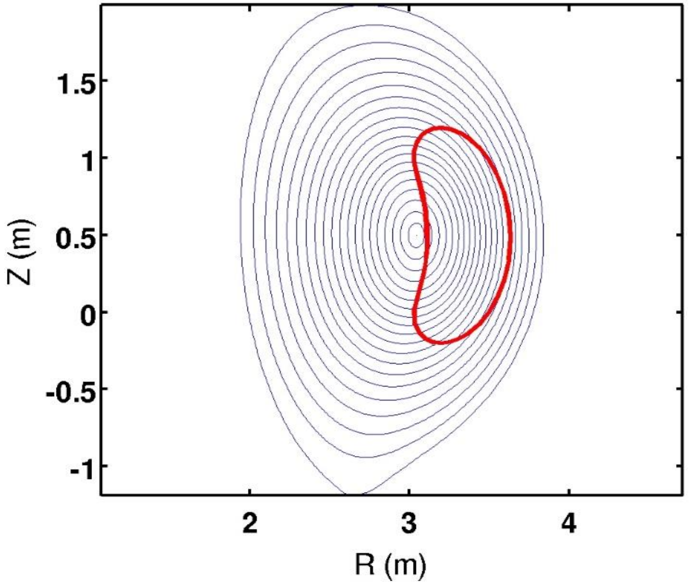
Energetic particle orbits in a tokamak

- Energetic (or fast) ions with v >> v_{th} have $\tau_b/t_{coll} << 1$

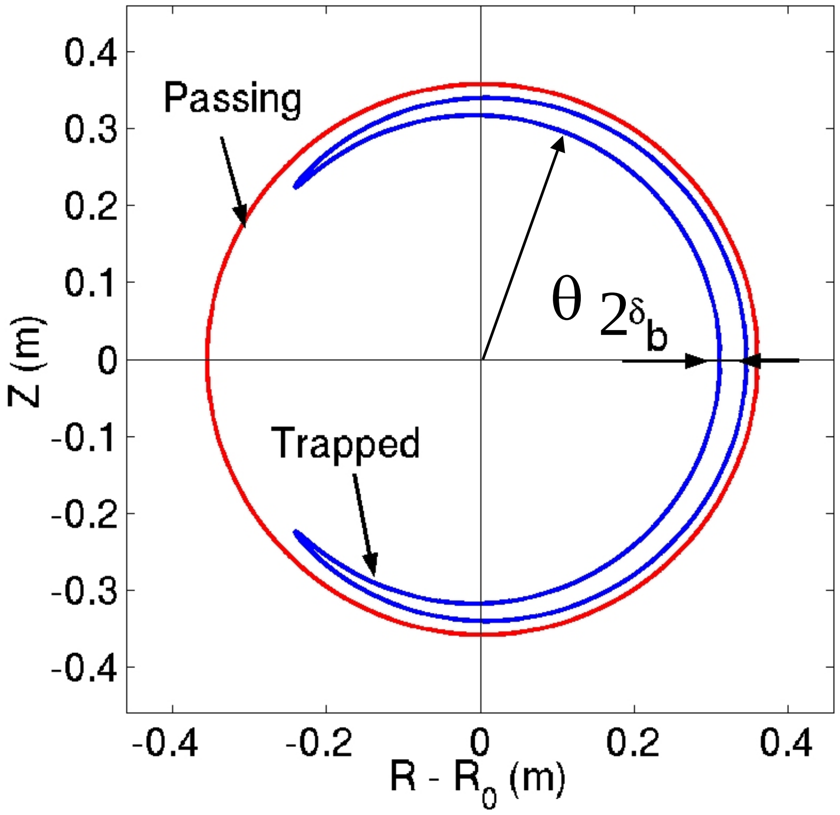


Guiding centre orbit





Standard, small width, banana orbits

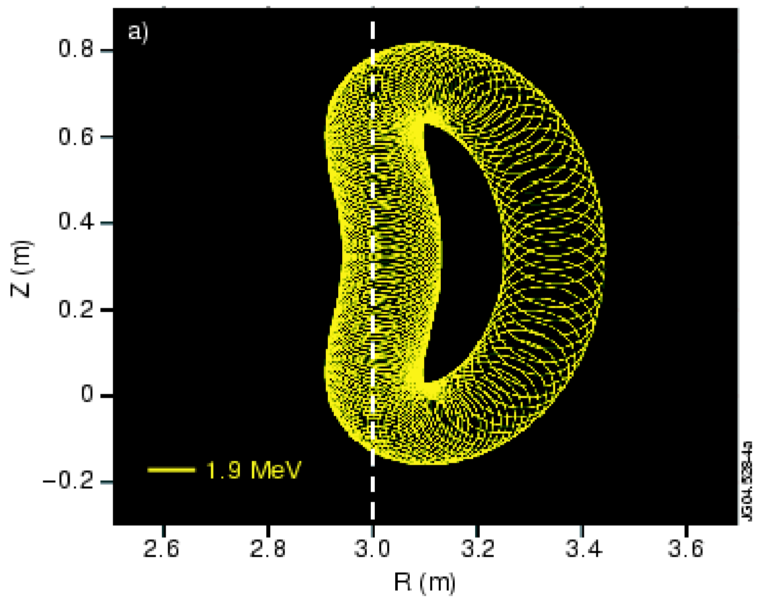


$$\delta_b \sim \frac{v_{\parallel 0}}{\omega_{B_\theta}} = \frac{v}{\omega_c} q \left(\frac{r}{R}\right)^{-1/2}$$

- Take 3.5 MeV alpha particle; q=1; r/R=0.1
- ITER $2\delta_b/a$ ~0.2
- JET $2\delta_b/a$ ~0.6

Non-standard orbits when $2\delta_b \gtrsim r$

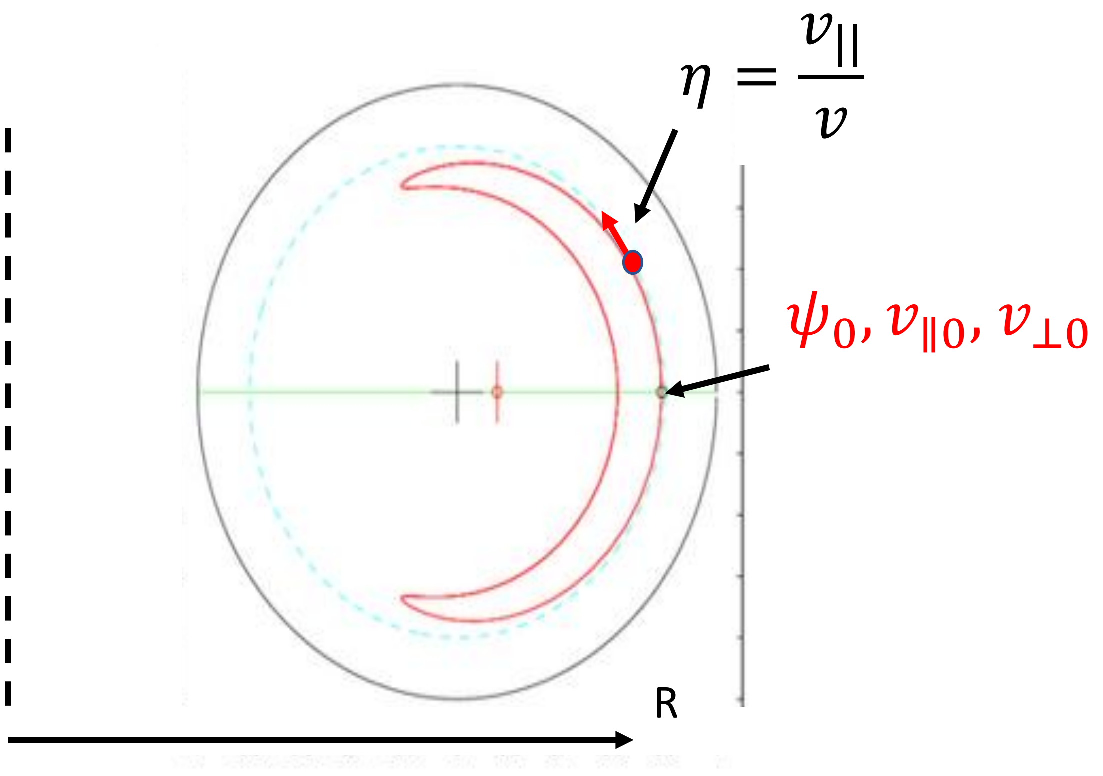
1.9 MeV alpha particle in JET: $2\delta_b \approx 0.4m$



Crossed border into non standard regime

Invariants

- How many variables, invariants, are needed to identify an orbit?
- Answer: three! E.g.
  1. Label to position along the orbit where $B = B_{min}$ by the poloidal flux function $\psi = \psi_0$
  2. The flux ψ_0 together with the parallel and perpendicular velocities, $v_{\|0}$ and $v_{\perp 0}$, then defines and orbit



The poloidal flux function is $\psi = \int_0^r RB_\theta dr$ (integration along midplane)

An often used set of Invariants

$W = \frac{1}{2}mv^2$ or v Energy or Velocity

$\Lambda = \frac{\mu B_0}{E} = \frac{v_\perp^2}{v^2}\frac{B_0}{B}$ Magnetic momentum / Energy

$P_\varphi = mRv_{||}\frac{B_\varphi}{B} + Ze\psi$ Toroidal canonical (or angular) momentum

ψ is the poloidal flux function (for a circular tokamak $\psi = \int_0^r RB_\theta dr$)

- There are regions in (E, Λ, P_φ) corresponding to two orbits, which we distinguish by $\sigma = \pm 1$
- Note that for trapped ions: $P_\varphi = Ze\psi\big|_{v_\parallel=0}$

26/006/2023
12th ITER International School June 2023

Orbit equation (approximate)

- Equating $v_{||}$ from P_φ with $v_{||} = \pm v\sqrt{1 - \Lambda B/B_0}$ yields

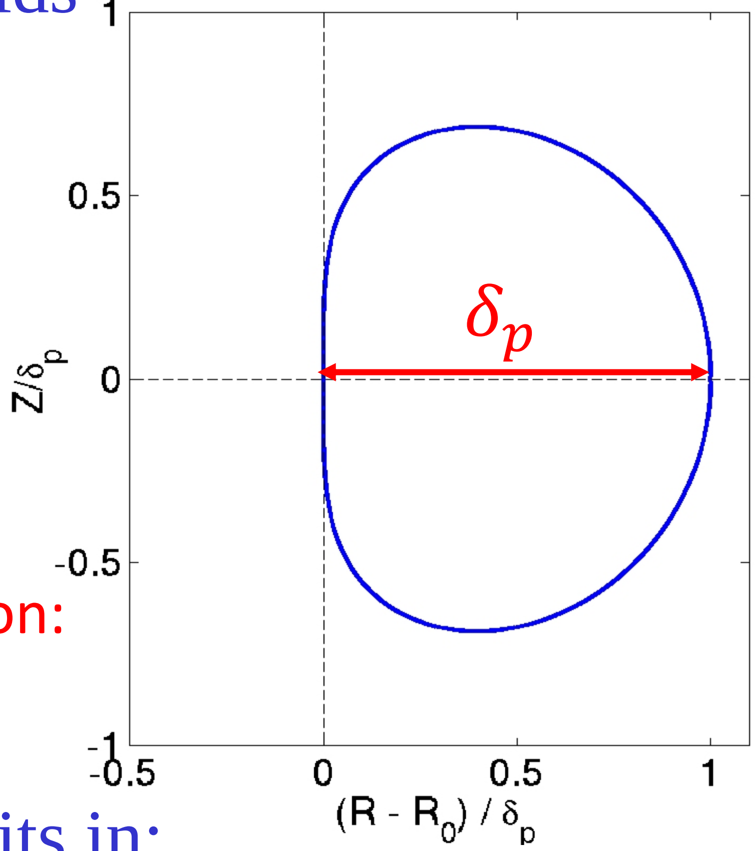
$$\pm\sqrt{1 - \Lambda \frac{B}{B_0}} \approx \frac{1}{mR_0 v}\left[-Ze\psi + P_\varphi\right]$$

- Consider a typical "Potato orbit" with $v_{||} = 0$ at the magnetic axis, → $\Lambda = 1, P_\varphi = 0$

$$\delta_p \approx \left(\frac{2qv}{R_0\omega_c}\right)^{2/3} R_0$$

Typical for a JET 1 MeV hydrogen ion:

$\delta_p = 30$ cm



- The equation was the basis for the classification of orbits in:

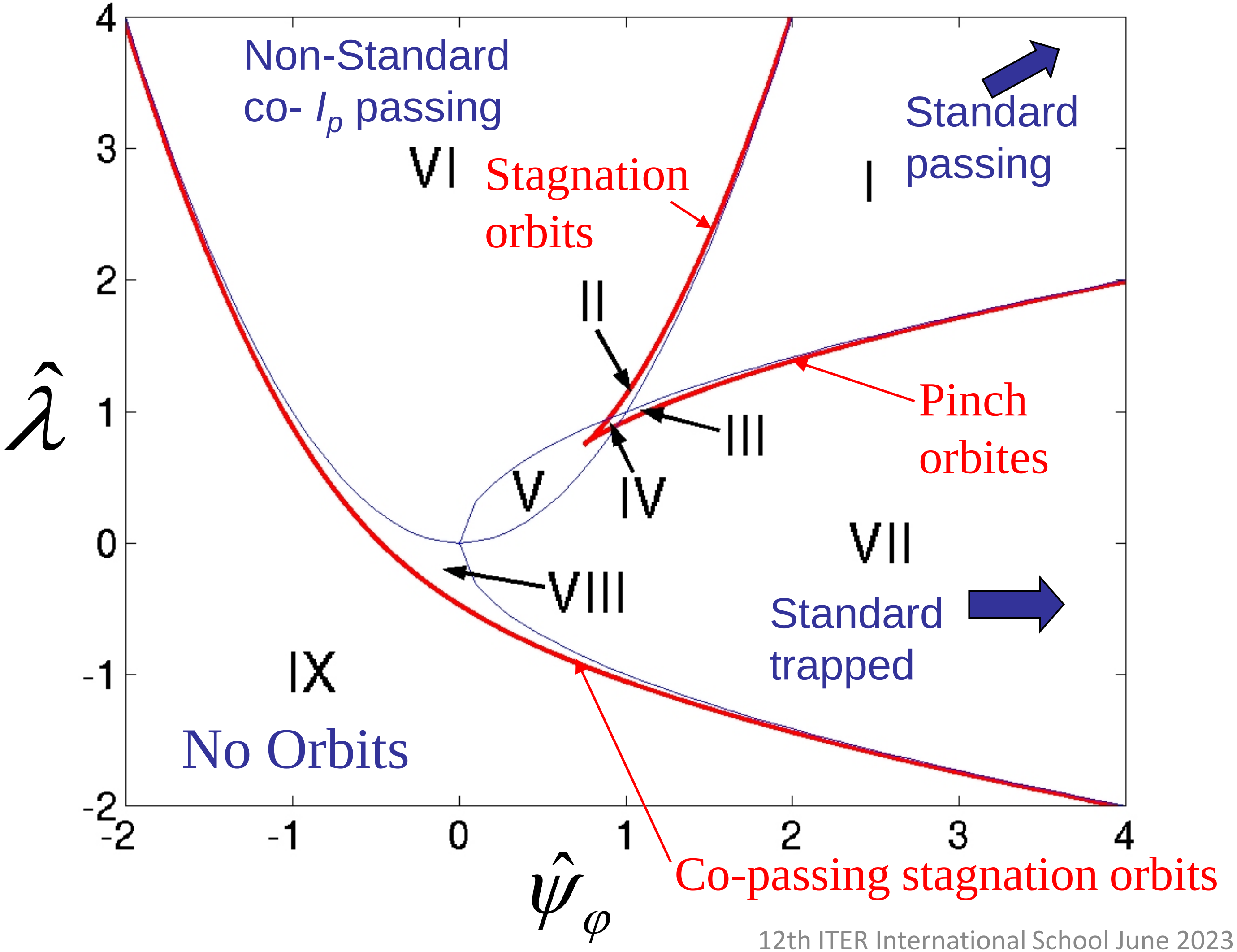
L.-G. Eriksson and F. Porcelli, PPCF, **43**, R145 (2001)

- Several authors have classified orbits see e.g. J. Egedal 2000 Nucl. Fusion **40** 1597

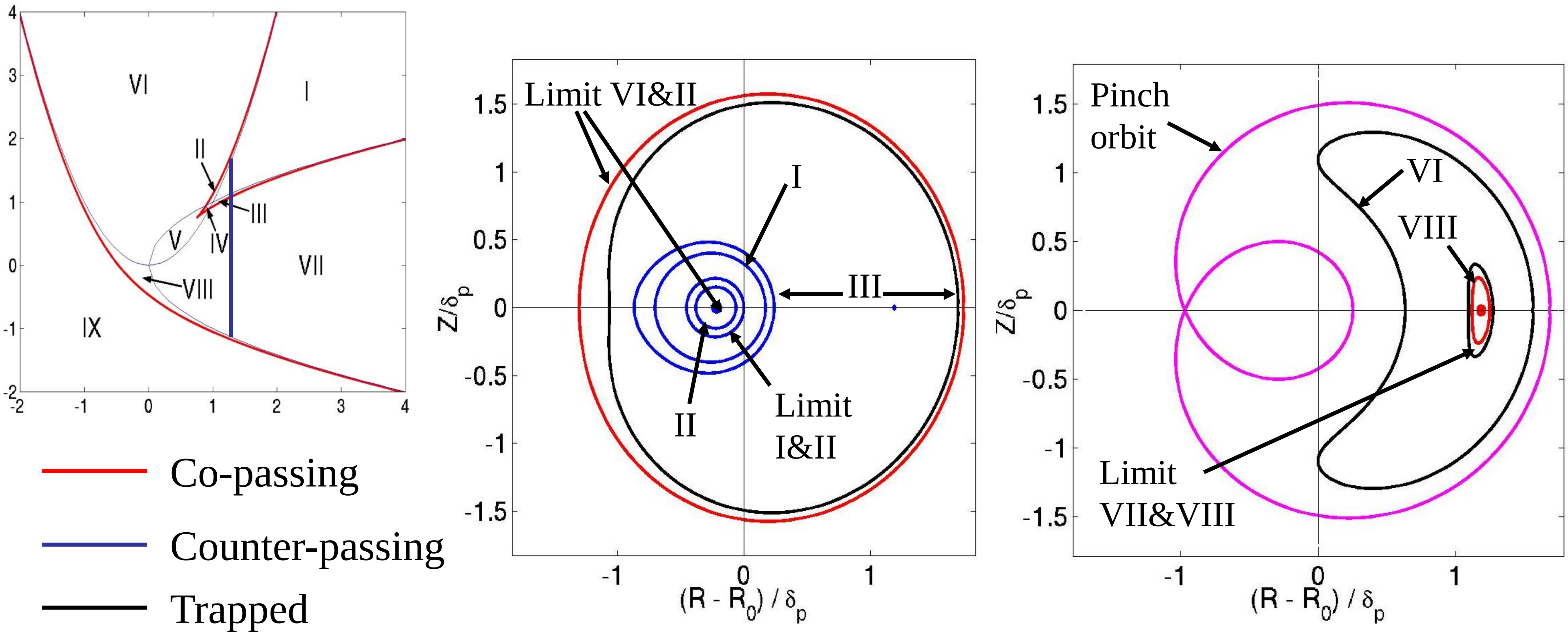
Orbit classification

Low β tokamak, circular flux surfaces; $\hat{\lambda} = \left(1 - \frac{1}{\Lambda}\right)\frac{R_0}{\delta_p}, \quad \hat{\psi}_\varphi = \frac{2q}{ZeB_0\delta_p^2} P_\varphi$

Reg.	Orbits
I	two
II	two
III	two
IV	two
V	one
VI	one
VII	one
VIII	one
IX	none



Orbits for $\hat{\psi}_\varphi = 1.2$



The orbit averaged Fokker-Planck equation

- In order to analyse fast ions we will need the Fokker-Planck equation

$$\vec{z} = (\vec{r}, \vec{v})$$

Collision operator

Source term

Loss term

$$\frac{\partial f}{\partial t} + \dot{z}^i \frac{\partial f}{\partial z^i} = C(f) + S - L(f) + Q(f)$$

Particle orbits enter here

Wave particle interaction

- We are mainly interested in evolution of f on the collisional time scale
- It is possible to reduce this 6D F-P equation to 3D by orbit averaging:

Intermediate variables $\vec{z} = (\vec{r}, \vec{v}) \rightarrow (\vec{J}, \vec{\theta})$

- The motion of a single particle in an axisymmetric torus is integrable → there exist a canonical transformation to action angle variables [1] $(\vec{J}, \vec{\theta})$
- In these the (unperturbed) Hamiltonian H_0 depends only on the actions:

$$H_0 = H_0(\vec{J}) \quad \Rightarrow \quad \Omega^i = \dot{\theta}_0^i = \frac{\partial H_0}{\partial J^i}$$

$$J^1 = \frac{m\mu}{Ze} = \frac{\Lambda E}{\omega_{c0}}, \qquad J^3 = P_\varphi = mRv_{||}\frac{B_\varphi}{B} + Ze\psi, \qquad J^2 \sim$$ Toroidal flux enclosed by a poloidal orbit

- Roughly speaking the angles describe:

θ^1 Position in the Larmor rotation

θ^2 Position along the poloidal guiding centre orbit

θ^3 Toroidal position of banana centre

$$\Omega^1 = \langle \omega_c \rangle$$

$$\Omega^2 = 2\pi/\tau_b$$

$$\Omega^3 = \langle \dot{\varphi} \rangle$$

$\langle \cdots \rangle =$ Orbit average

[1]A.N. Kaufman Phys. Fluids. 1972.

- Fokker-Planck equation after variable transformation $\vec{z} = (\vec{r}, \vec{v}) \rightarrow (\vec{J}, \vec{\theta})$

$$\frac{\partial f}{\partial t} + \dot{\theta}^i \frac{\partial f}{\partial \theta^i} = C(f) + S - L + Q(f)$$

- Multiple time scale expansion: $f = f_0 + \frac{\tau_b}{\tau_c} f_1 + \cdots$
- To order -1 we have: $\Omega^i \frac{\partial f_0}{\partial \theta^i} = 0 \Rightarrow f_0 = f_0(\vec{J}) = f_0(\vec{I})$
- Define orbit average: $\langle \cdots \rangle = (2\pi)^{-3} \iiint_0^{2\pi} (\cdots)\, d^3\theta \approx \frac{1}{\tau_b} \int_0^{\tau_b} (\cdots) d\tau$
- The zero order equation yields after applying $\langle \cdots \rangle$

$$\frac{\partial f_0}{\partial t} = \langle C(f_0) \rangle + \langle S \rangle - \langle L \rangle + \langle Q(f_0) \rangle$$

- The collision operator is conservative, i.e. it is represented as a divergence of a flow in phase space.

$$C(f) = \frac{1}{v^2}\frac{\partial}{\partial v}\left\{v^2\left[\alpha(v)f + \beta(v)\frac{\partial f}{\partial v}\right]\right\} + \frac{\partial}{\partial \eta}\left[\gamma(v)(1-\eta^2)\frac{\partial f}{\partial \eta}\right]$$

Slowing down; Energy diffusion; Pitch angle scattering

- For test particles it simplifies remarkably in the high energy limit $v >> v_{th}$

Slowing down on el.; Slowing down on ions

$$C(f) = C_{s.d.}(f) + C_{p.a.s}(f) = \frac{1}{v^2}\frac{\partial}{\partial v}\left[v^2\frac{v + \frac{v_c^3}{v^2}}{t_s}f\right] + \frac{\partial}{\partial \eta}\left[\frac{1}{t_s}\frac{v_\gamma^3}{4v^3}(1-\eta^2)\frac{\partial f}{\partial \eta}\right]$$

Slowing down term; Pitch angle scattering

t_s Is the ion electron slowing down time; v_c is the critical velocity; v_γ is a characteristic velocity for pitch angle scattering

Slowing down time and characteristic energies of $C(f)$

CHALMERS

- Example DT plasma $n_e = 1 \cdot 10^{20} m^{-3}$

Slowing down time:

$$t_s \approx 6.27 \times 10^{14} \frac{A T_{e,eV}^{3/2}}{Z^2 n_e ln\Lambda}$$

Critical Energy:

$$\frac{1}{2} m v_c^2 = 14.8 k T_e A \left[\frac{1}{n_e} \sum_j \frac{n_j Z_j^2}{A_j} \right]^{2/3}$$

Char. Energy for pitch angle scatt.:

$$\frac{1}{2} m v_\gamma^2 = 14.8 k T_e \left[2 A^{1/2} Z_{eff} \right]^{2/3}$$

	D		^{4}He	
	T_e = 10 keV	T_e = 20 keV	T_e = 10 keV	T_e = 20 keV
t_s	0.7s	2s	0.35s	1s
$\frac{1}{2} m v_c^2$	165 keV	330 keV	330 keV	660 keV
$\frac{1}{2} m v_\gamma^2$	296 keV	592 keV	373 keV	746 keV

For further details see e.g. T.H Stix Nuclear Fusion **15**, 737 (1975)

High energy orbit averaged $C(f_0)$

- Using tensor transformation rules, we obtain for the invariants $\vec{I} = (v, \Lambda, P_\varphi)$

$$\langle C(f_0)\rangle = \frac{1}{\sqrt{g}}\frac{\partial}{\partial v}\left[\sqrt{g}\left\langle\frac{v+\frac{v_c^3}{v^2}}{t_s}\right\rangle f_0\right] + \frac{1}{\sqrt{g}}\frac{\partial}{\partial P_\varphi}\left[\sqrt{g}\left\langle mR\frac{B_\varphi}{B}\eta\frac{v+\frac{v_c^3}{v^2}}{t_s}\right\rangle f_0\right] +$$

$$\frac{1}{\sqrt{g}}\frac{\partial}{\partial I^i}\left[\sqrt{g}\left\langle\frac{1}{t_s}\frac{v_\gamma^3}{4v^3}(1-\eta^2)\frac{\partial I^i}{\partial \eta}\frac{\partial I^j}{\partial \eta}\right\rangle\frac{\partial f_0}{\partial I^j}\right]$$

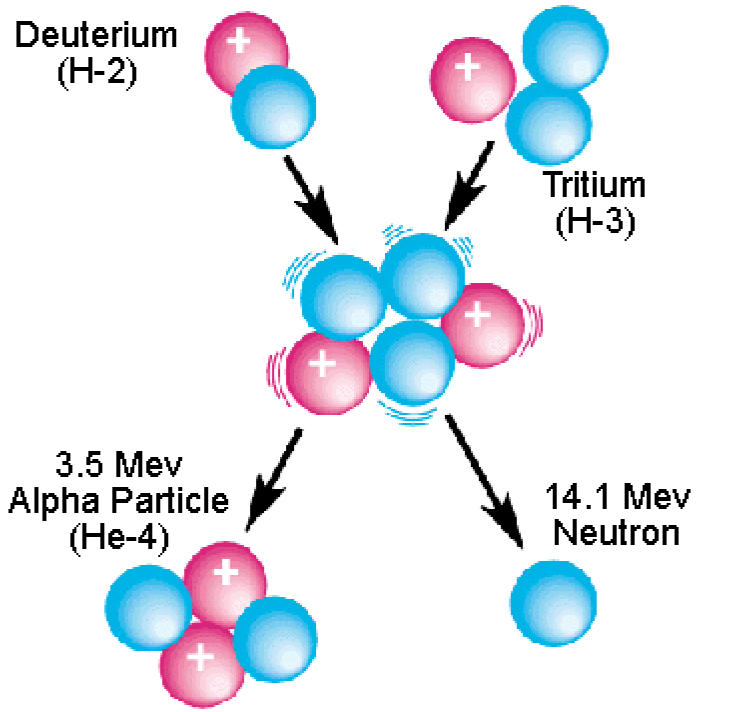
$$\sqrt{g} = \left|\frac{\partial \vec{z}}{\partial(\vec{I},\vec{\theta})}\right| = \frac{v^3\tau_b}{4\pi m\omega_{c0}}$$

Methods for solution of the orbit averaged Fokker-Planck equation

- Direct solution with finite difference scheme (very intricate and impressive)
  - *F. S. Zaitsev; M. R. O'Brien; M. Cox Physics of Fluids B: Plasma Physics* 5, 509–519 (1993).
  - Yu V Petrov and R W Harvey 2016 *Plasma Phys. Control. Fusion* **58** 115001 (including QL operator; very impressive)
- Direct solution with a Monte Carlo algorithm
  - *Carlsson J., et al Proceedings of the Joint Varenna-Lausanne Workshop "Theory of Fusion Plasmas", Bologna: Editrice Compositori, 1994, p. 351. (FIDO code)*
- Orbit following Monte Carlo codes with an acceleration scheme
  - A. Pankin, D. McCune, R. Andre et al., Computer Physics Communications Vol. 159, No. 3 (2004) 157-184. (NUBEAM used in TRANSP)
  - Seppo Sipilä *et al* 2021 *Nucl. Fusion* **61** 086026 (ASCOT code augmented with ICRF module)

Alpha particles and classical slowing down

- Alpha particles are born in D-T fusion reactions:
  D+T → He^4 (3.52 MeV) + n (14.06 Mev).
- The alpha particles slows down by collisions with the bulk plasma, and should sustain the plasma temperature and thereby the fusion burn in a reactor.
- Hence, the confinement of alpha particles is crucial.
- To analyse fusion alpha particles with with the Fokker-Planck equation we need the source: $S(\vec{r},\vec{v})$



Deuterium-Tritium Fusion Reaction

- The spectrum of the alpha particle source depends on the distribution function of the reacting spices $S = S(f_D, f_T)$
- The kinematics of the alpha particle producing process dictates*:

$$E_\alpha = \frac{1}{2} m_\alpha V^2 + \frac{m_n}{m_n + m_\alpha}(Q + K) + \mathrm{V}\sin\varphi \sqrt{\frac{2 m_n m_\alpha}{m_n + m_\alpha}(Q + K)}$$

$$Q \approx 17.49\ MeV$$

$$K = \frac{1}{2} \frac{m_D m_T}{m_D + m_T} (\vec{v}_D - \vec{v}_T)^2$$

V is the centre of mass velocity of the reacting D and T ions

φ is the angle between $\vec{v}_\alpha$ and $\vec{V}$ in the centre of mass frame

*see e.g. H Brysk. Plasma Physics,15(7):611, 1973

- If the deuterium and tritium ions have thermal distributions (i.e. are Maxwell distributed) the source term can be approximated by*,

$$\langle S \rangle = \frac{\langle \sigma v \rangle_{DT}}{c_{norm}} e^{-\frac{5m_\alpha^2 (v^2 - v_{\alpha 0}^2)^2}{64 T_i E_{\alpha 0}}}$$

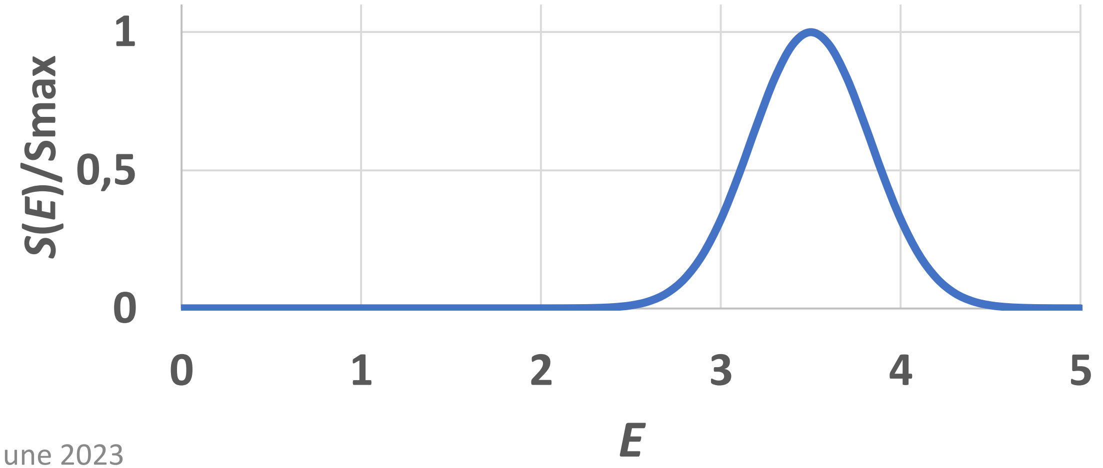
$$C_{norm} = \int_0^\infty e^{-\frac{5m_\alpha^2 (v^2 - v_{\alpha 0}^2)^2}{64 T_i E_{\alpha 0}}} 4\pi\, v^2 dv$$

$$E_{\alpha 0} = \frac{1}{2} m_\alpha v_{\alpha 0}^2 = 3.5 MeV$$

- The full width at half maximum FWHM of the alpha particle source, $\Delta E_{\alpha FWHM}$, is: $\Delta E_{\alpha FWHM}(MeV) = 0.088\sqrt{T_i(KeV)}$

- Typical for ITER would be $T_i = 20\ keV$

➡ $\Delta E_{\alpha FWHM} = 0.39\ MeV$

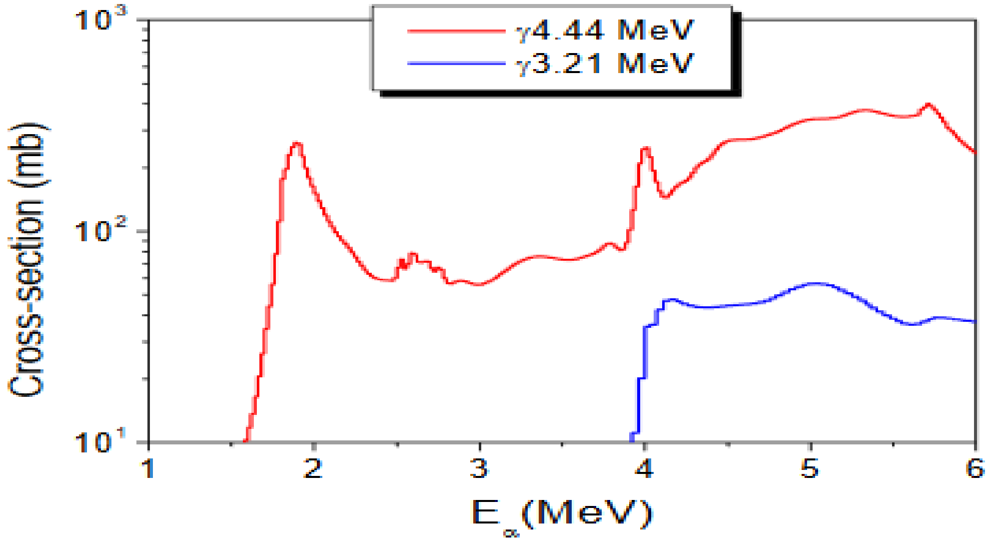


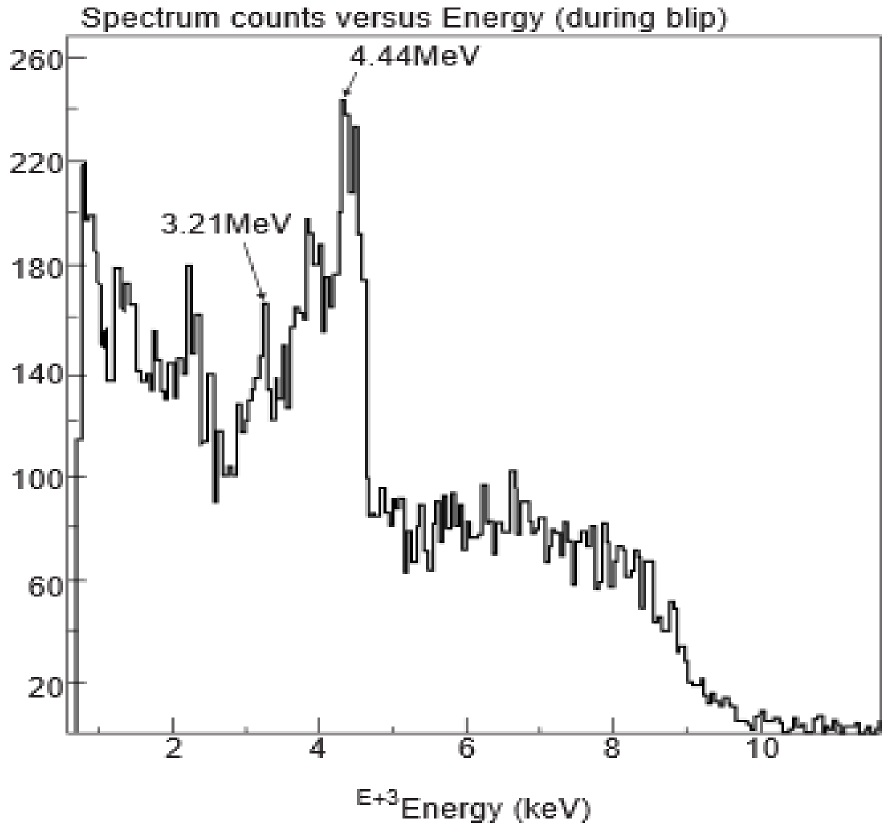
*see e.g. H Brysk. Plasma Physics,15(7):611, 1973

Υ ray measurement at JET during trace tritium campaign in 2003

Interaction between α's and Be impurity → excitation of C^{13} and following de-excitations to Υ rays

Cross section for gamma rays





Deuterium plasma with 15 MW of NBI injected at 105 KeV and 1.5 MW of Tritium blip; $T_i(0)$ = 6 keV and $n_e = 6 \cdot 10^{10}$ m^{-3}

Kiptily et al. Phys Rev Letter 93(2004) 115001

- To make analytic progress, we turn to he simplest form of the Fokker-Planck equation: small banana width and an isotropic alpha particle source

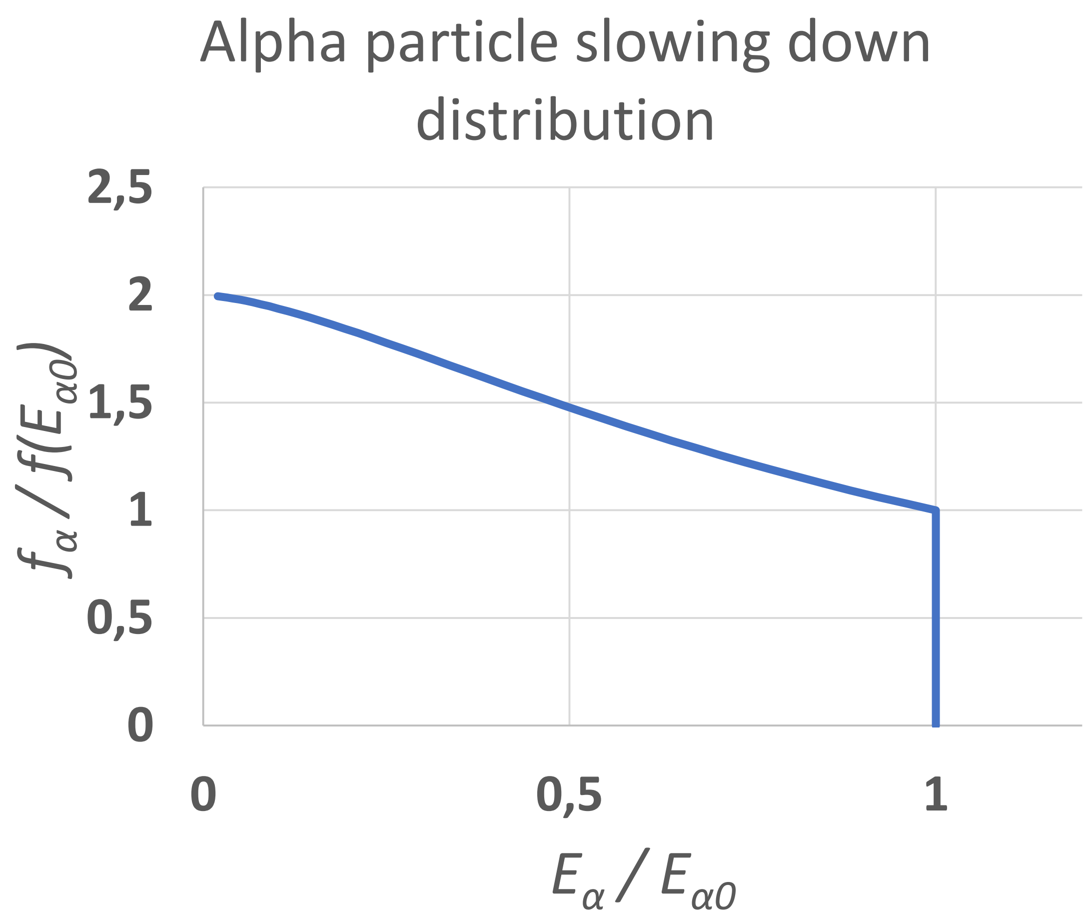
$$\frac{\partial f_{\alpha 0}}{\partial t} = \frac{1}{v^2}\frac{\partial}{\partial v}\left[v^2 \frac{v + \frac{v_c^3}{v^2}}{t_s} f_\alpha\right] + \frac{\langle \sigma v\rangle_{DT}}{4\pi v^2}\delta(v_\alpha - v_{\alpha 0})$$

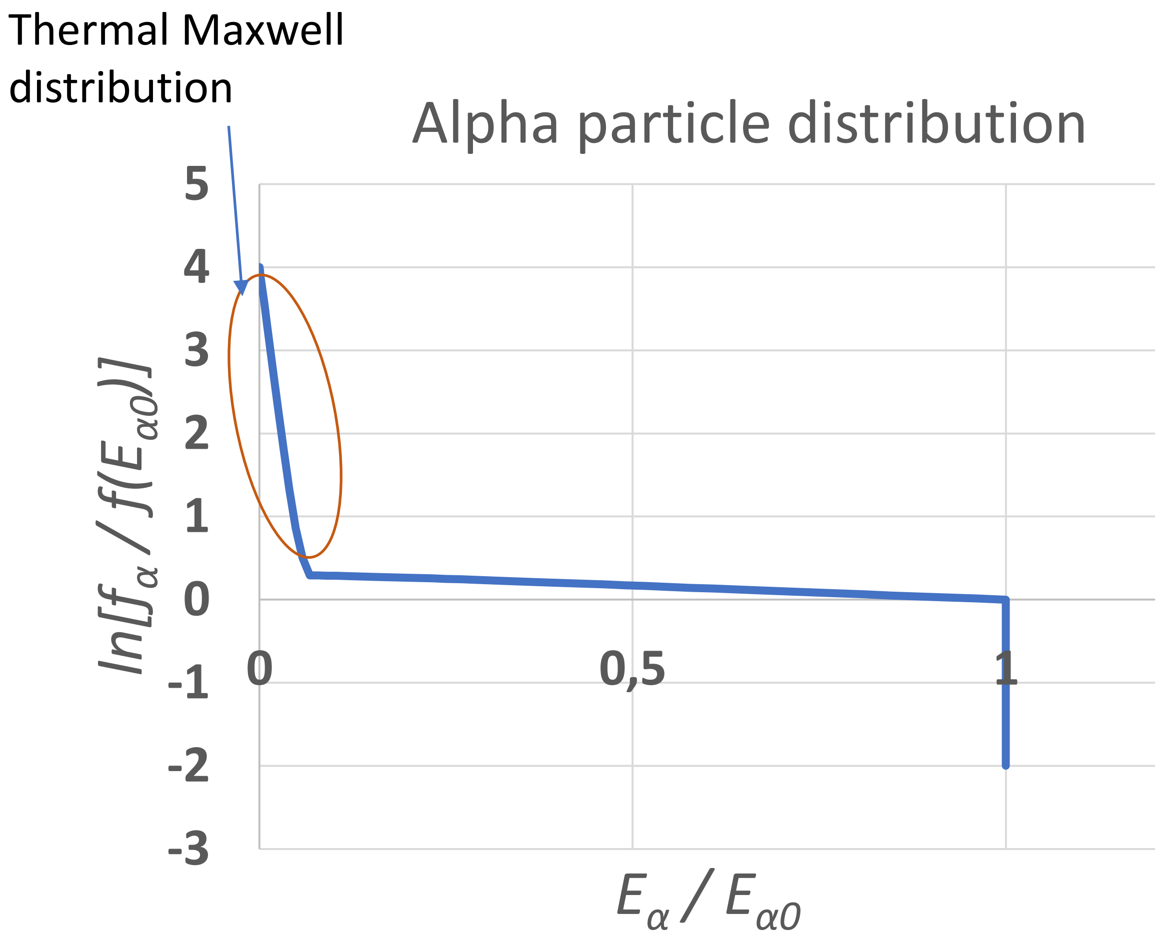
- In steady state the equation is easy to integrate and we get,

$$f_{\alpha 0}(v, \vec{r}) = \frac{t_s \langle \sigma v\rangle_{DT}}{4\pi v_{\alpha 0}^2}\frac{H(v - v_{\alpha 0})}{v^3 + v_c^3}$$

(*H* is the Heaviside step function)

- This is the classical slowing down distribution


Alpha particle slowing down distribution
2,5
2
1,5
1
0,5
0
0
0,5
1
$f_\alpha / f(E_{\alpha 0})$
$E_\alpha / E_{\alpha 0}$


Thermal Maxwell distribution
Alpha particle distribution
5
4
3
2
1
0
-1
-2
-3
0
0,5
1
$ln[f_\alpha / f(E_{\alpha 0})]$
$E_\alpha / E_{\alpha 0}$

- The collisional power density to the electrons is given by

$$p_e = -\int_0^\infty \frac{m_\alpha}{2} v^2 C_{s.d,e}(f_{\alpha 0}) 4\pi v^2 dv = -\int_0^\infty \frac{m_\alpha}{2} v^2 \frac{1}{v^2}\frac{\partial}{v}\left(v^2 \frac{v}{t_s} f_{\alpha 0}\right) 4\pi v^2 dv_\alpha$$

- With the classical slowing down distribution the integral is doable,

$$p_e = p_\alpha \left[1 - \frac{2v_c^2}{3v_{\alpha 0}^2}\left\{\frac{1}{2}\ln\left[1 - \frac{v_{\alpha 0}}{v_c} + \left(\frac{v_{\alpha 0}}{v_c}\right)^2\right] - \ln\left(1 + \frac{v_{\alpha 0}}{v_c}\right) + \sqrt{3}\left[arctan\left(\frac{\frac{2v_{\alpha 0}}{v_c} - 1}{\sqrt{3}}\right) + \frac{\pi}{6}\right]\right\}\right]$$

With $T = 20\ keV$ ($\frac{1}{2}mv_c^2 \approx 660keV$) we obtain

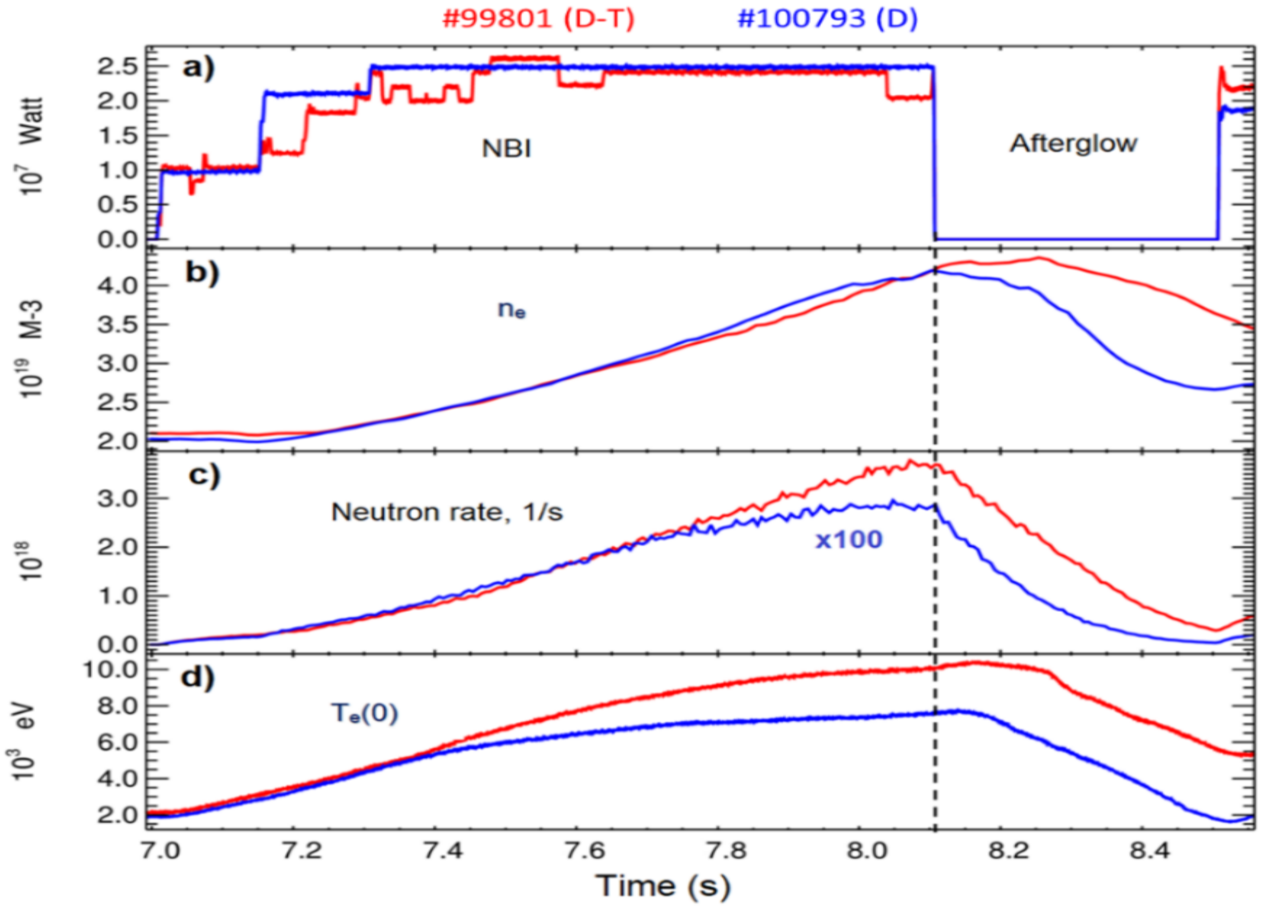
$p_e \approx 0.7P_\alpha$ and $p_i \approx 0.3P_\alpha$

With $T = 10\ keV$ ($\frac{1}{2}mv_c^2 \approx 330keV$) we obtain

$p_e \approx 0.83p_\alpha$ and $p_i \approx 0.17P_\alpha$

Alpha particle heating of electrons in JET DTE2

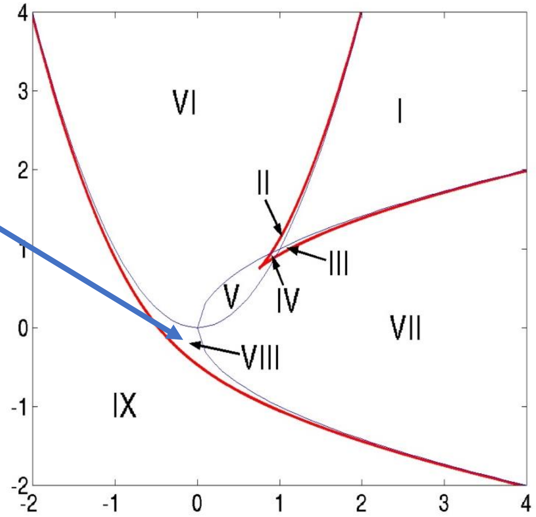
- Hot off the press1!
- "Afterglow" experiments in the recent JET DTE2 experiments demonstrated electron heating by alpha particles.
- The results are consistent with TRANSP simulations.



V. Kiptily et al, accepted in Physical Review Letters

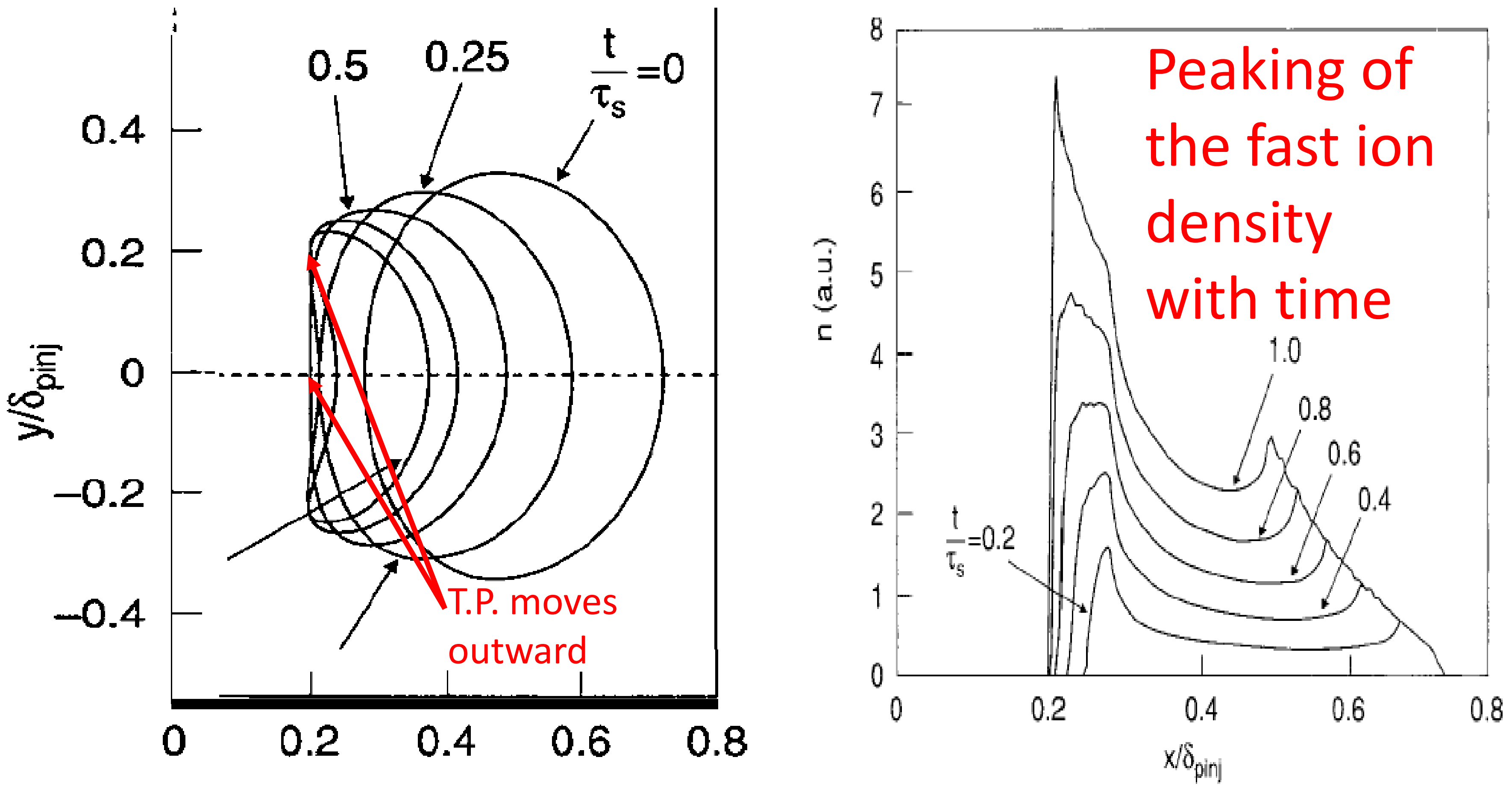
Neoclassical transport due to collisional drag

- For illustration we consider alpha particles borne at the same point in phase space, reg. VIII.
- We can see the solution as a Green's function



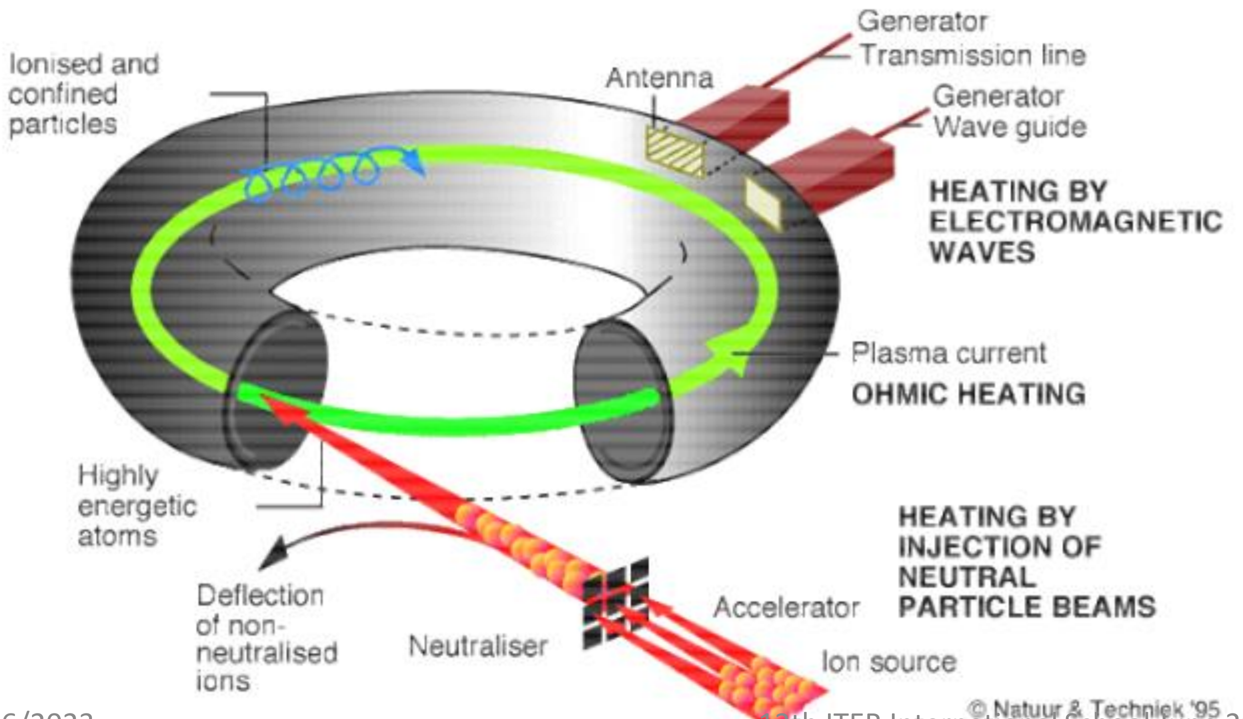
$$\frac{\partial f_0}{\partial t} = \frac{1}{\sqrt{g}}\frac{\partial}{\partial v}\left[\sqrt{g}\underbrace{\left\langle\frac{v}{t_s}\right\rangle}_{-\Gamma_c^v} f_0\right] + \frac{1}{\sqrt{g}}\frac{\partial}{\partial P_\varphi}\left[\sqrt{g}\underbrace{\left\langle\frac{P_\varphi - Ze\psi}{t_s}\right\rangle}_{-\Gamma_c^{P_\varphi}} f_0\right]$$
$$+\bar{S}_0\delta(v - v_0)\delta(\Lambda - \Lambda_0)\delta\left(P_\varphi - P_{\varphi 0}\right)$$

Orbit Evolution and fast ion density



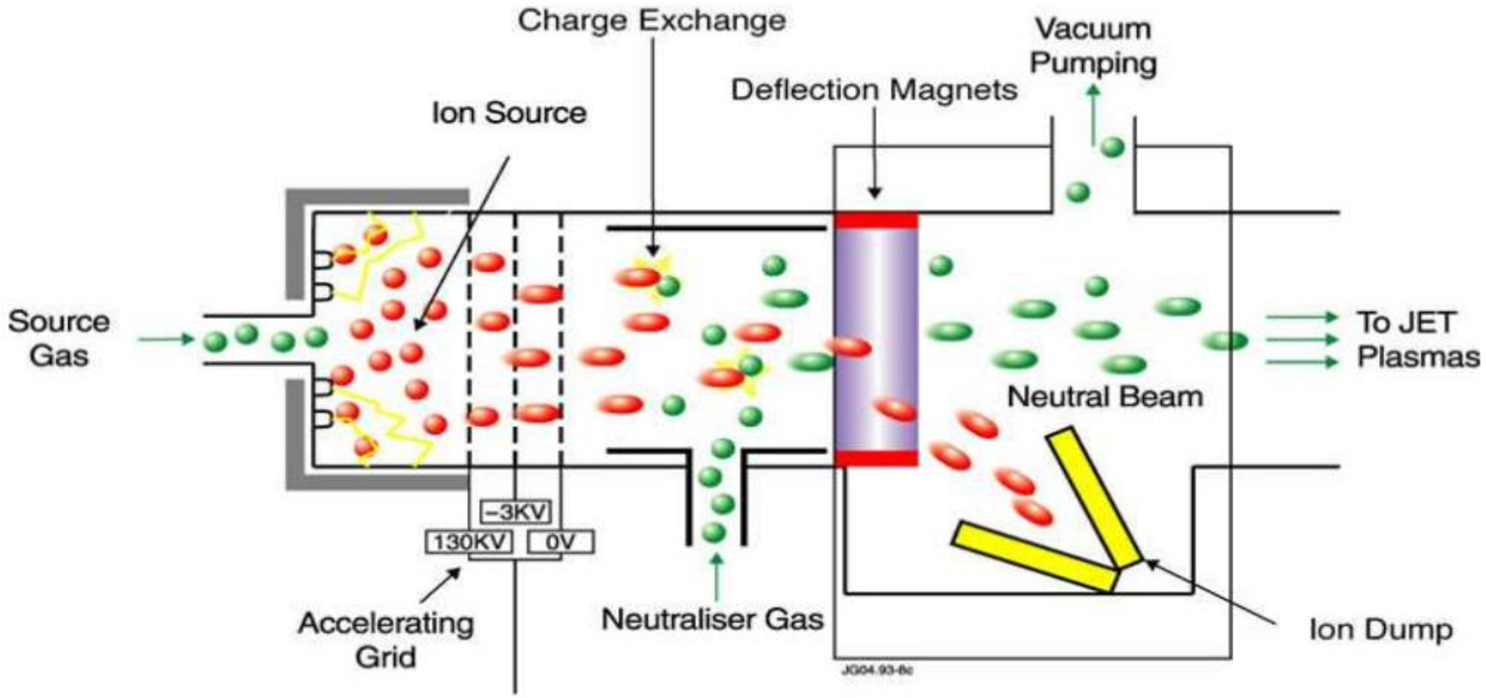
Neutral Beam Injection

- Neutral Beam Injection has been the work horse heating system for many present day tokamaks.
- One the beam particles enter the plasma it is a very robust heating method.
- Like alpha particles, NBI heats the bulk plasma via collisional slowing down



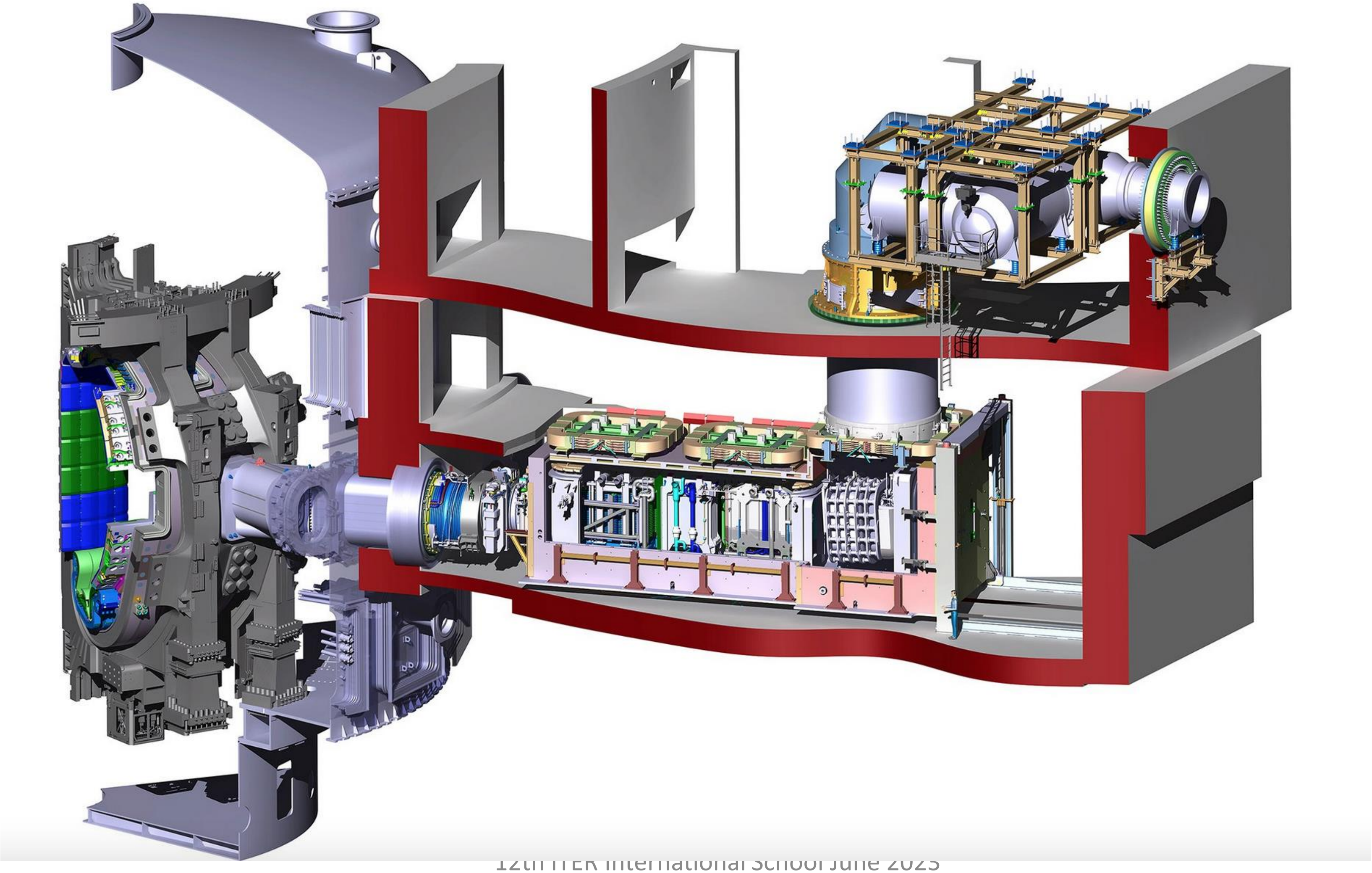
- There are two possibilities:
  - Positive ion sources
  - Negative ion sources
- For injection energies of the order 200 keV and above, the neutralisation efficiency of positive ions is low → negative ion sources are necessary

NBI principle



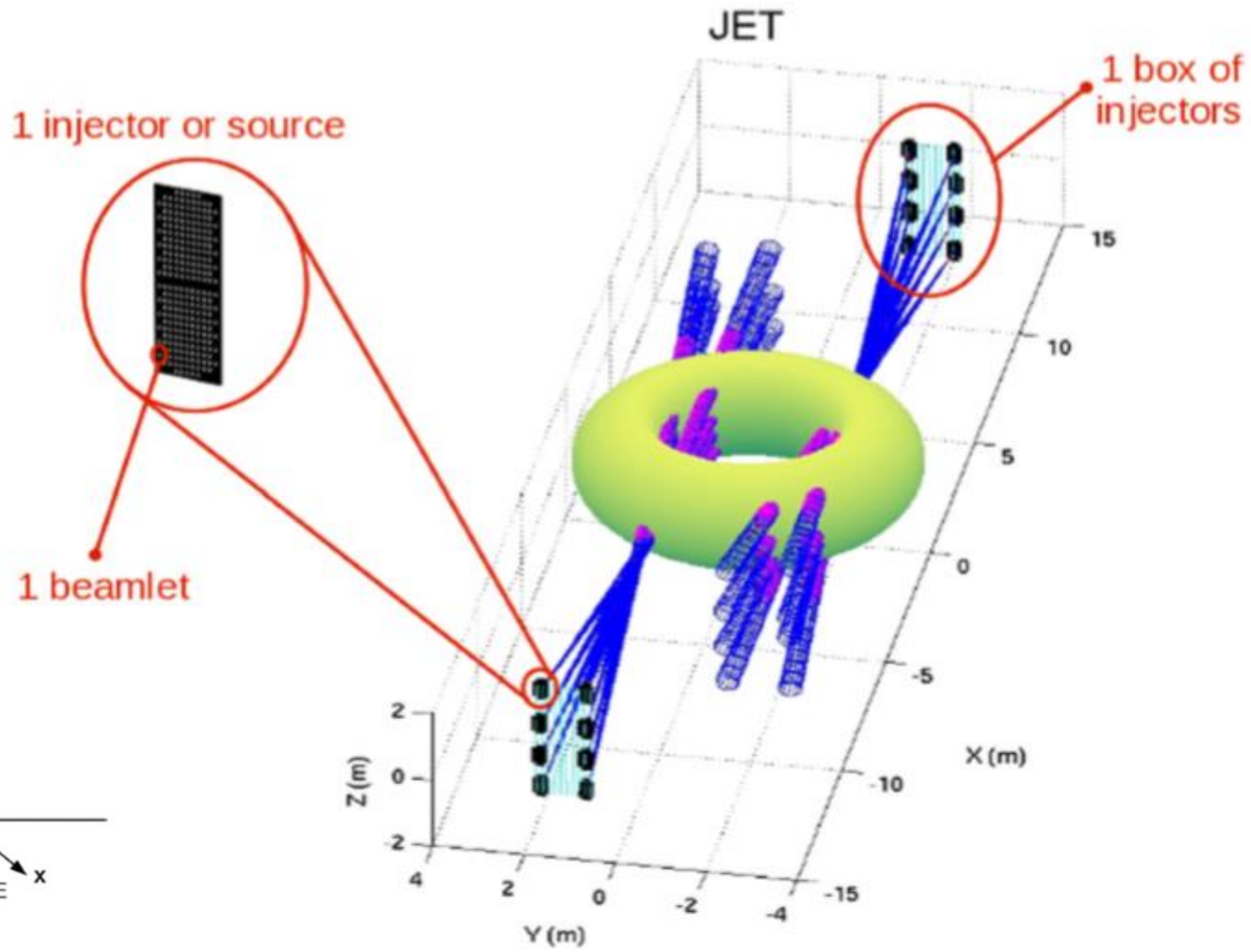
- Note that for positive ion sources, molecular ions (D2 , D3) will be in the mix →
- After ionisation in the plasma
  - Full the full injection energy
  - 1/2 injection energy
  - 1/3 injection energy
- Negative ion sources only yield injected ions at the full energy

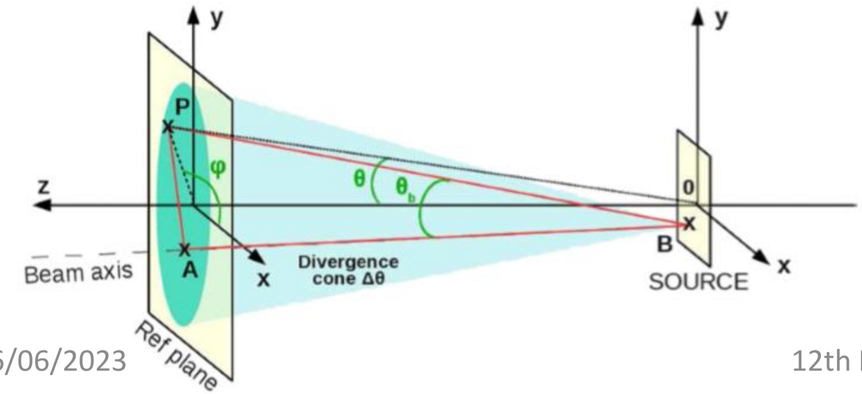
1 Mev negative ion Injector for ITER



NBI deposition calculation

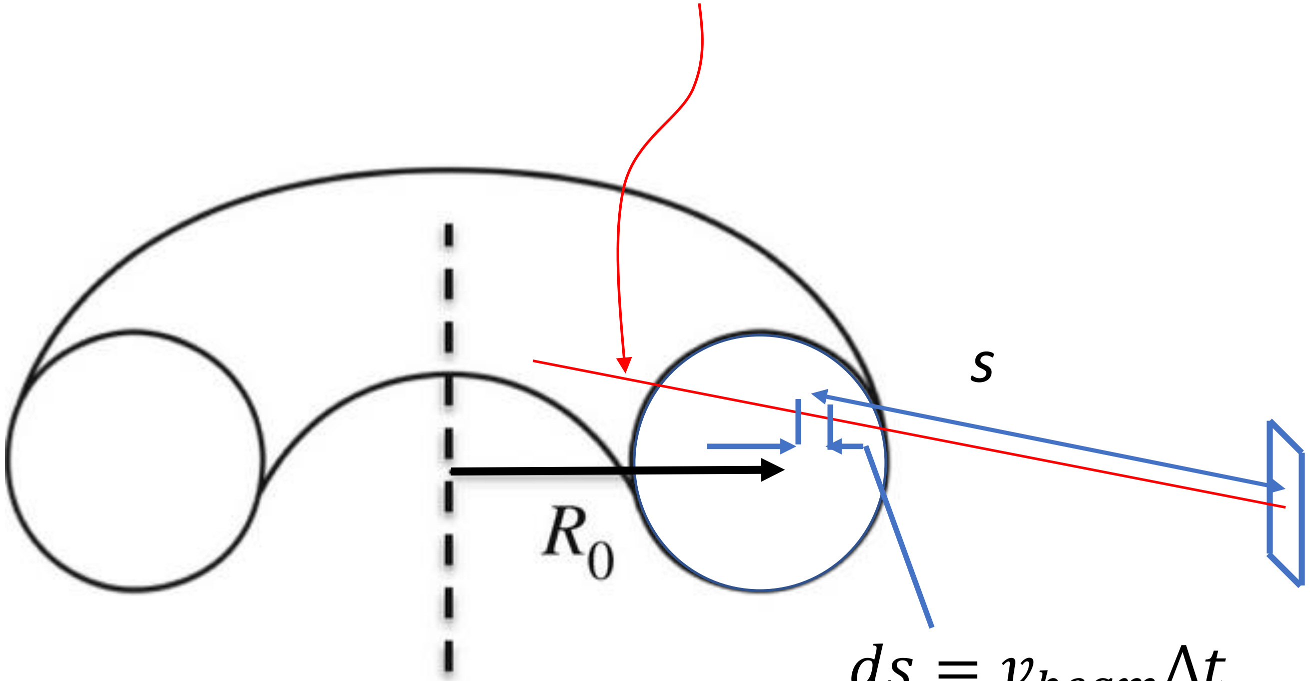
- In order to calculate deposition profile of NB particles we have to consider:
  - Ionisation rate along injection path
  - Beam divergence
  - Shine trough of the NB particles





CHALMERS

- Beam attenuation along a line



$$ds = v_{beam}\Delta t$$

$$I = I_0 e^{-\int_0^s \frac{ds}{\lambda}}$$

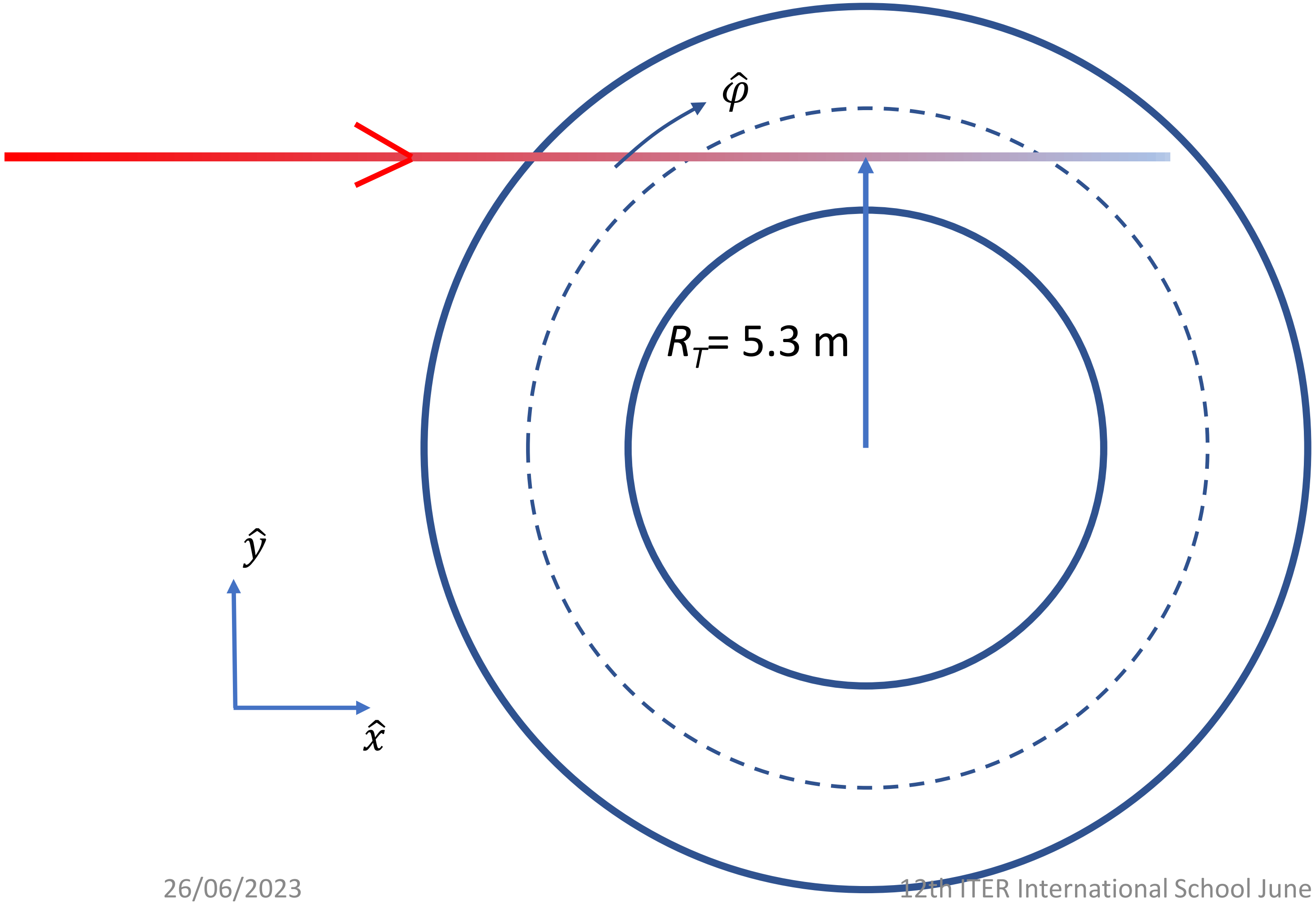
$$v_{rel} = |\vec{v}_{beam} - v_i|$$

$$\frac{1}{\lambda} = \frac{\langle \sigma_e v_e \rangle n_e}{v_{beam,m}} + \frac{\langle (\sigma_{cx} + \sigma_i) v_{rel} \rangle n_i}{v_{beam,m}} + \sum_{\substack{imp. \\ j}} \sigma_j n_j$$

- σ_{ion} and σ_e are the stopping cross-sections collisions with plasma ions and electrons, respectively.
- σ_{cx} is the charge exchange cross-section.
- These can e.g. be found in OPEN-ADAS
- For positive ion NBI "m" represents full, 1/2 and 1/3 energy

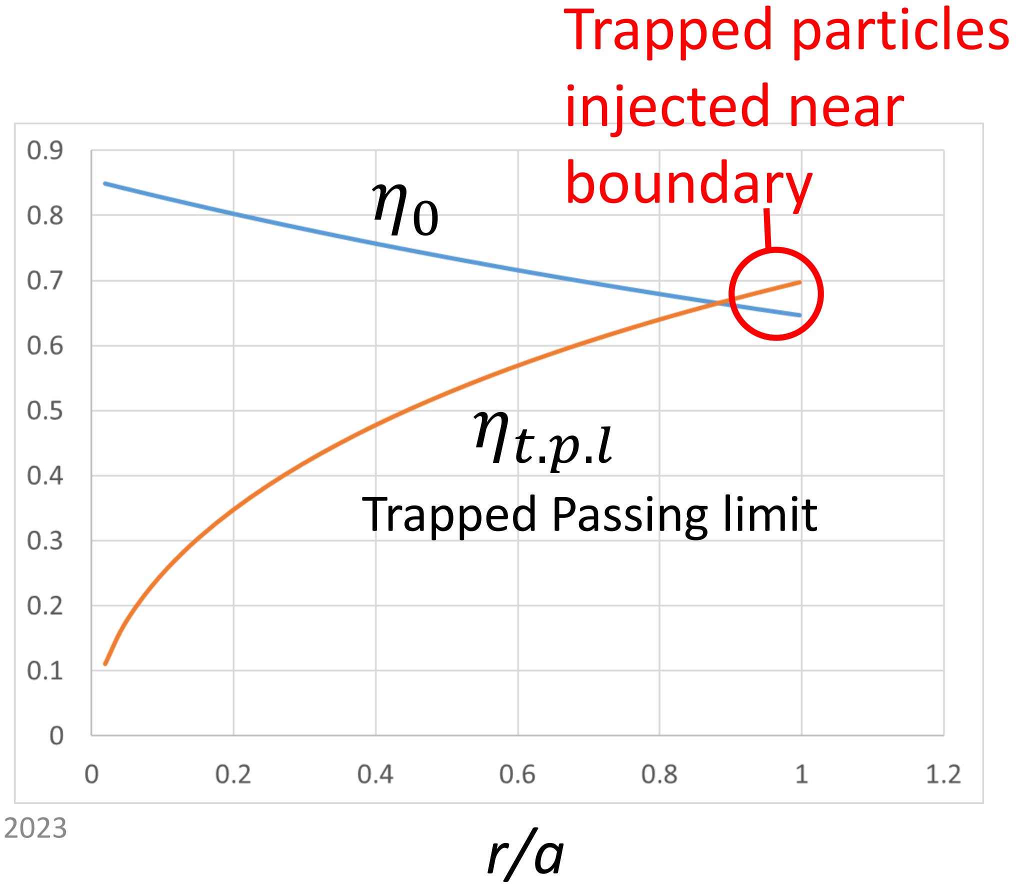
- Methods of calculating NBI deposition
  - Monte Carlo methods
    - NFREYA: R.H. Fowler, J.A. Holmes and J.A. Rome, NFREYA -- A Monte Carlo Beam Deposition Code for Non-circular tokamak Plasmas, Rep. ORNL-TM-6845, Oak Ridge National Laboratory, TN (1979).
    - Model in the TRANSP package NUBEAM
    - BBNBI in the ASCOT package O. Asunta et al Comp. Physcs Comm.188 (2015), 33
  - Narrow beam models
    - Y. Feng et al. Computer Physics Communications **88** (1995) 161
    - M. Schneider et al. Nucl. Fusion 51 (2011) 063019
  - Pencil model
    - P.M. Stubberfield and M.L. Watkins, Experimental department research note DPA(06)87, Multiple Pencil Beam, JET Joint Undertaking, Abingdon, Oxfordshire (1987).

Injection geometry in ITER (very simplified)



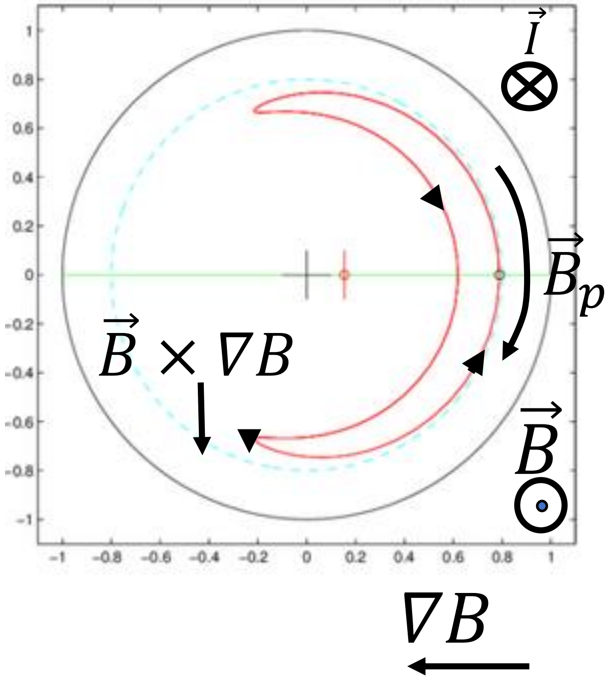
- The cosine of the pitch angle is roughly approximated by,

$$\eta_0 \approx \hat{x} \cdot \hat{\varphi} = \frac{R_T}{R}$$

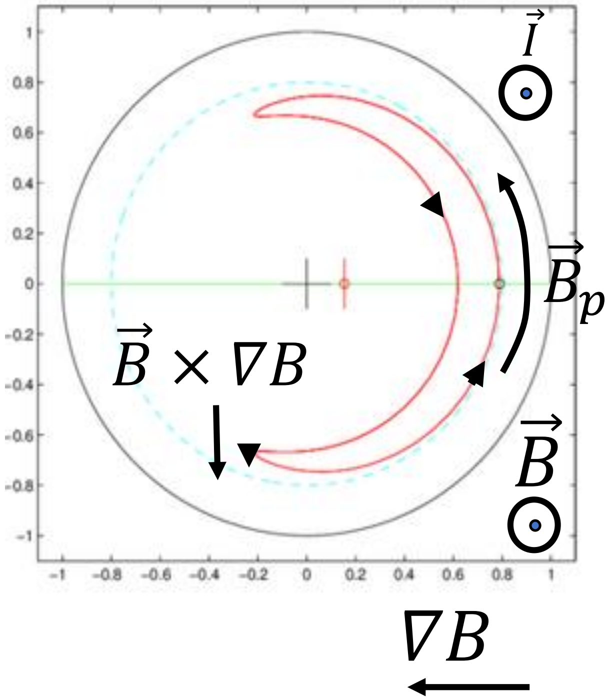


Interlude: in which direction is a trapped ion travelling on its outer leg?

- In which direction does an ion travel on the outer leg of a banana orbit?
- At the banana tip $\vec{v} = \cancelto{=0}{\vec{v}_{||}} + \vec{v}_d$



On the outer leg the ion is travelling opposite $\vec{B}$ i.e. in the co-current direction



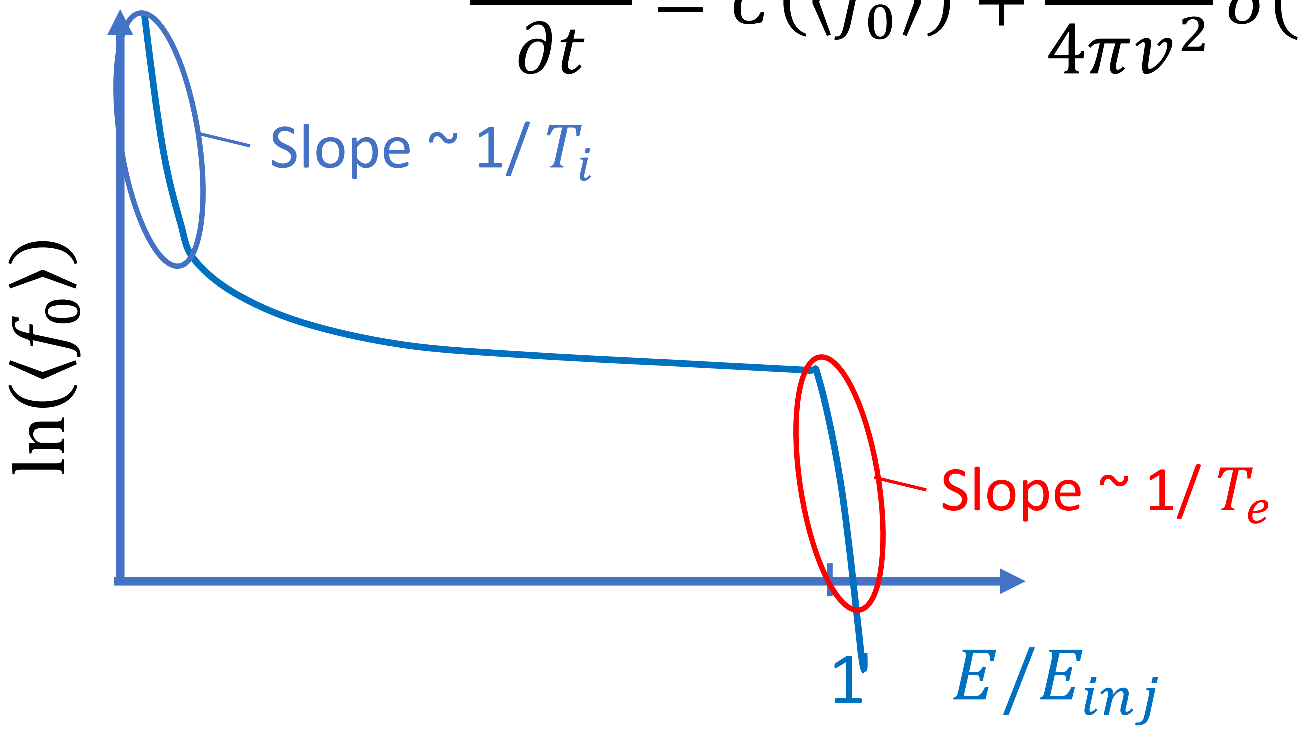
On the outer leg the ion is travelling along $\vec{B}$ i.e. in the co-current direction

- A trapped ion is always travelling in the co-current direction on the outer leg of its banana orbit
- In order to avoid fast ion losses near the edge, injection must therefore be in the co-current direction

NBI distribution function

- One can show that in the small banana width limit an appropriately defined pitch angle averaged distribution function $\langle f_0 \rangle$, satisfies

$$\frac{\partial \langle f_0 \rangle}{\partial t} = C(\langle f_0 \rangle) + \frac{S_0}{4\pi v^2}\delta(v - v_0)$$



- Full neoclassical calculation of NBI slowing down distribution with TRANSP has been found to be consistent with CTS measurements in ASDEX-Upgrade[1]

[1] S K Nielsen et al 2015 Plasma Phys. Control. Fusion 57 035009

- For assessing the collisional power transfer to bulk plasma ions and electrons we can use the same approximate formula as for alpha particles with $v_{\alpha 0}$ exchanged for v_{inj}

$$p_e = p_{NBI}\left[1 - \frac{2v_c^2}{3v_{\mathrm{inj}}^2}\left\{+\frac{1}{2}\ln\left[1 - \frac{v_{\mathrm{inj}}}{v_c} + \left(\frac{v_0}{v_c}\right)^2\right] - \ln\left(1 + \frac{v_{\mathrm{inj}}}{v_c}\right) + \sqrt{3}\left[arctan\left(\frac{\frac{2v_{\mathrm{inj}}}{v_c} - 1}{\sqrt{3}}\right) + \frac{\pi}{6}\right]\right\}\right]$$

- For D injection at $E_0 = 1MeV$ into a DT plasma with $n_e = 1 \cdot 10^{20} m^{-3}$:

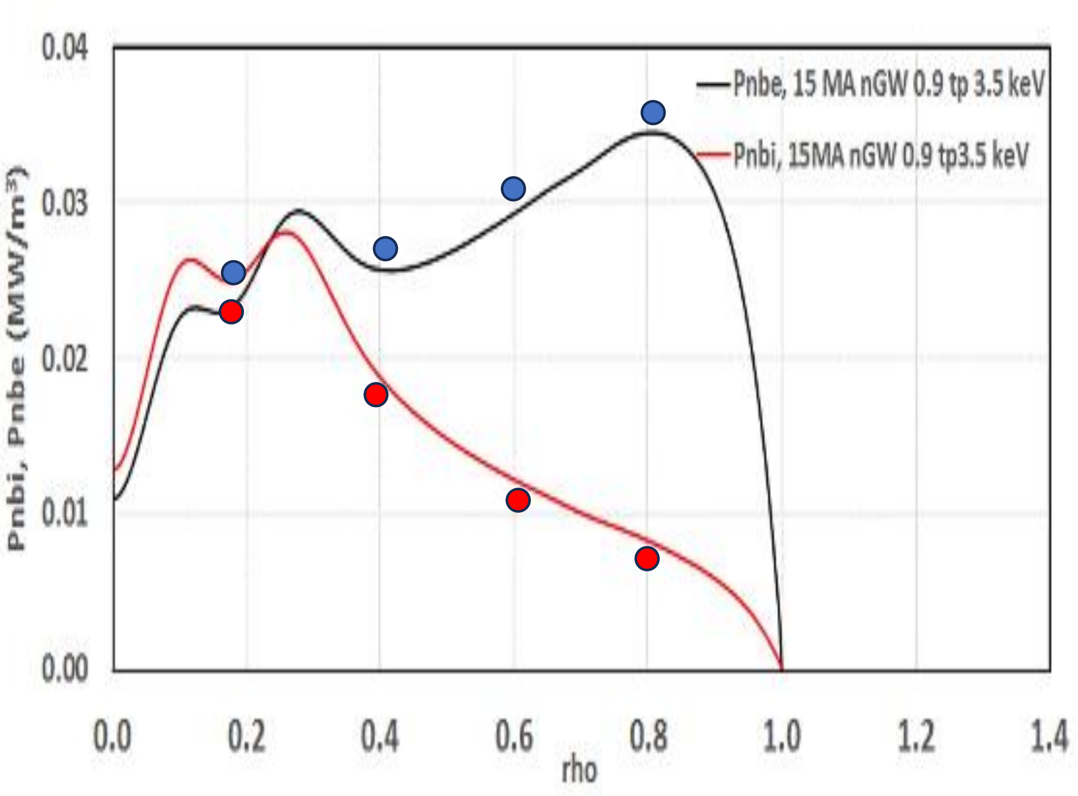
With $T = 25\ keV$ $(\frac{1}{2}mv_c^2 \approx 410keV)$ we obtain

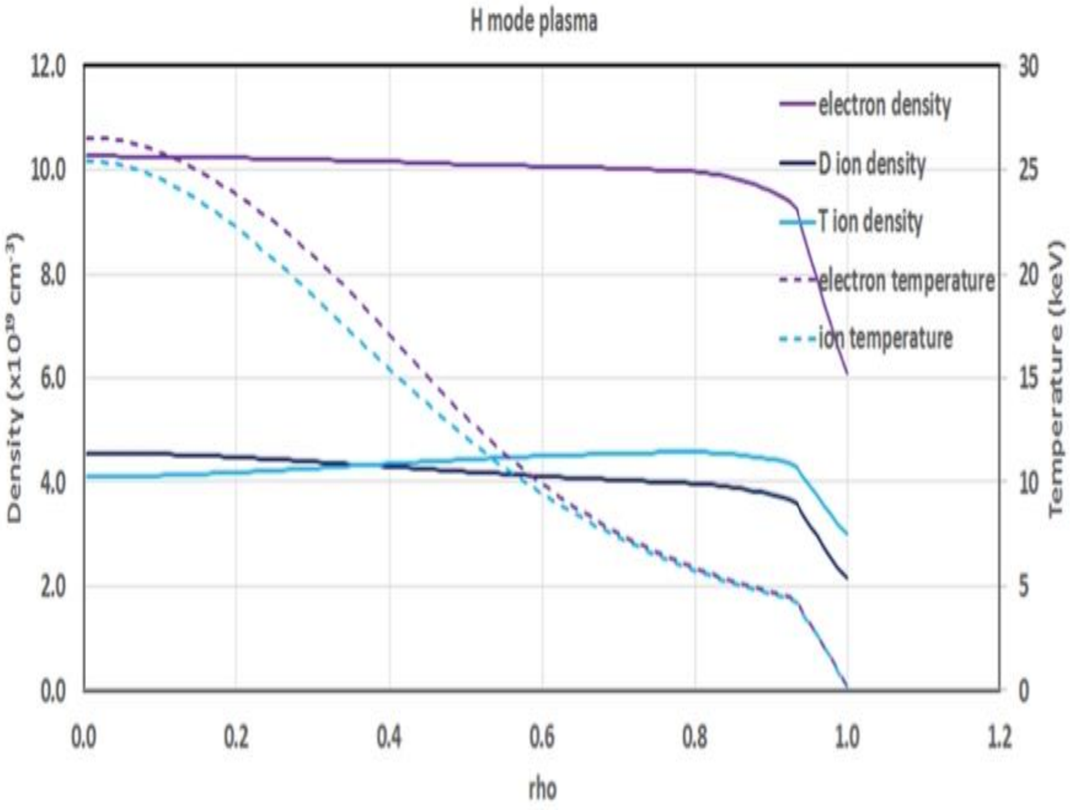
$p_e \approx 0.5p_{NBI}$ and $p_i \approx 0.5\ P_{NBi}$

With $T = 10\ keV$ $(\frac{1}{2}mv_c^2 \approx 165keV)$ we obtain

$p_e \approx 0.73p_{NBi}$ and $p_i \approx 0.27p_{NBi}$

- Simulated power density transferred to the plasma ions and electrons by 33 MW NBI for a 15 MA/5.3 T Q=10 ITER DT plasma



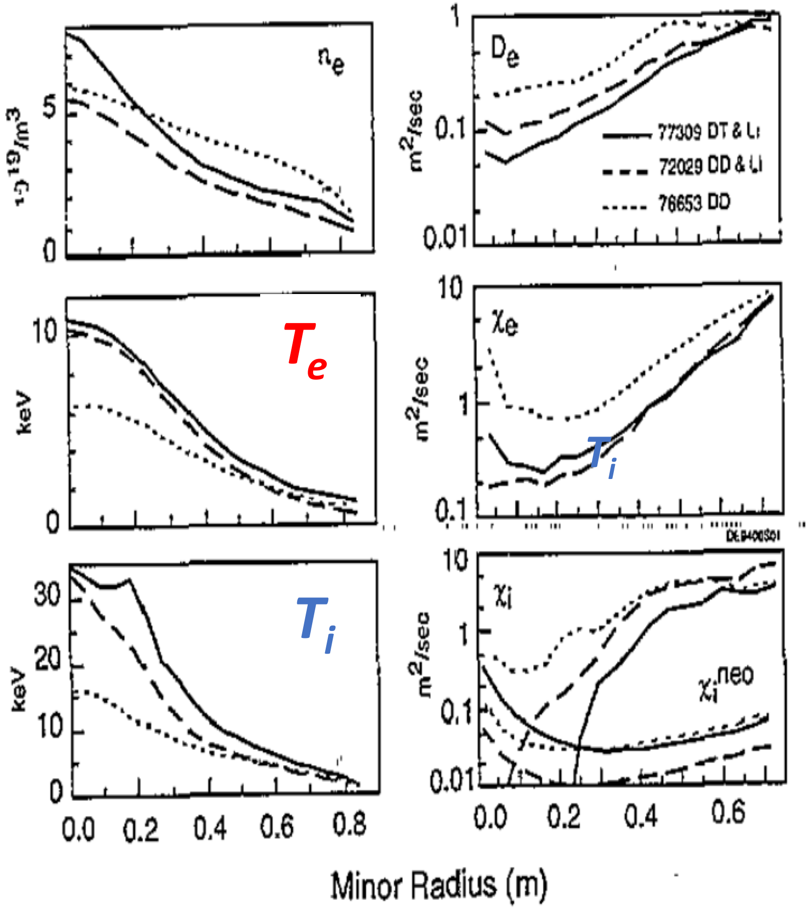


"Simple" formula

M J Singh et al. New J. Phys.**19** (2017)055004

- Deuterium tritium experiment with high NBI power in TFTR and JET

TFTR

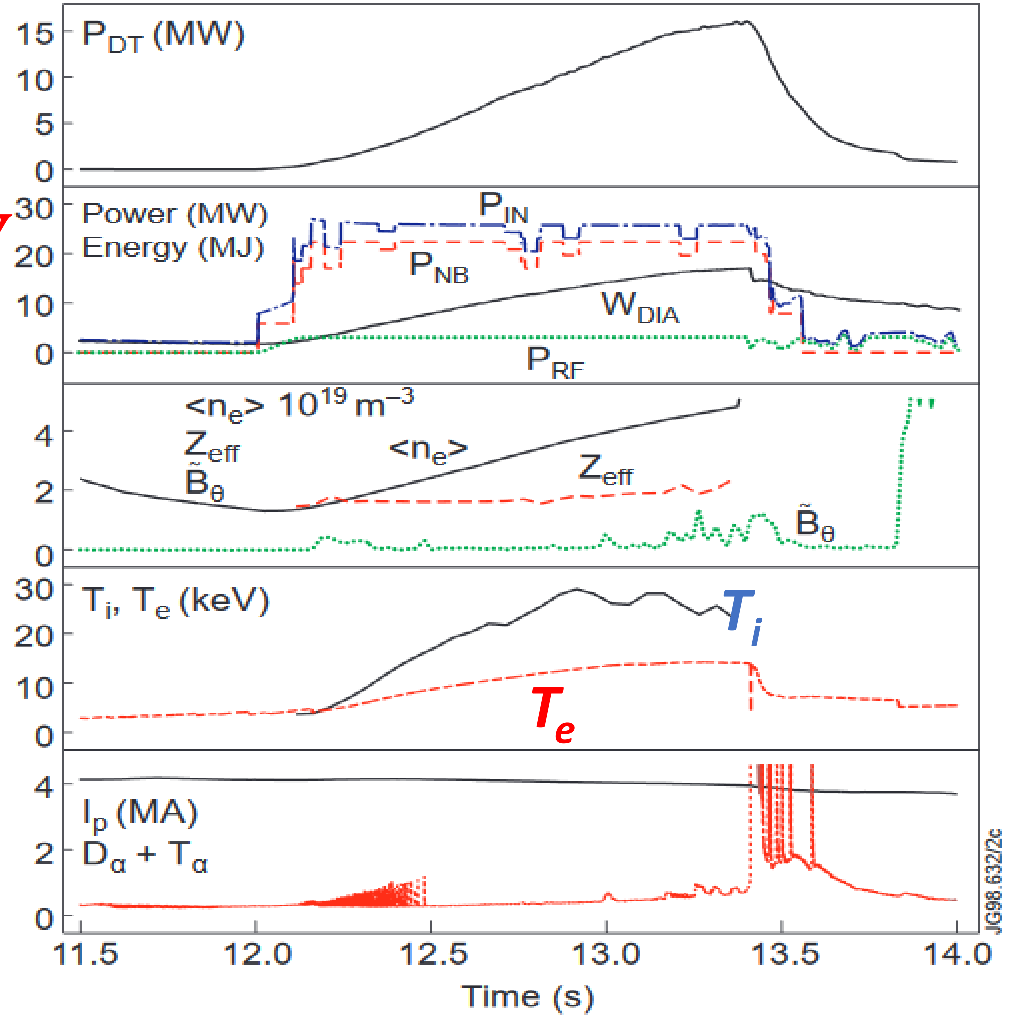


$E_{inj} \sim 100\ keV$

$\frac{1}{2} m v_c^2 \sim 170\ keV$

T_i significantly greater than T_e

JET

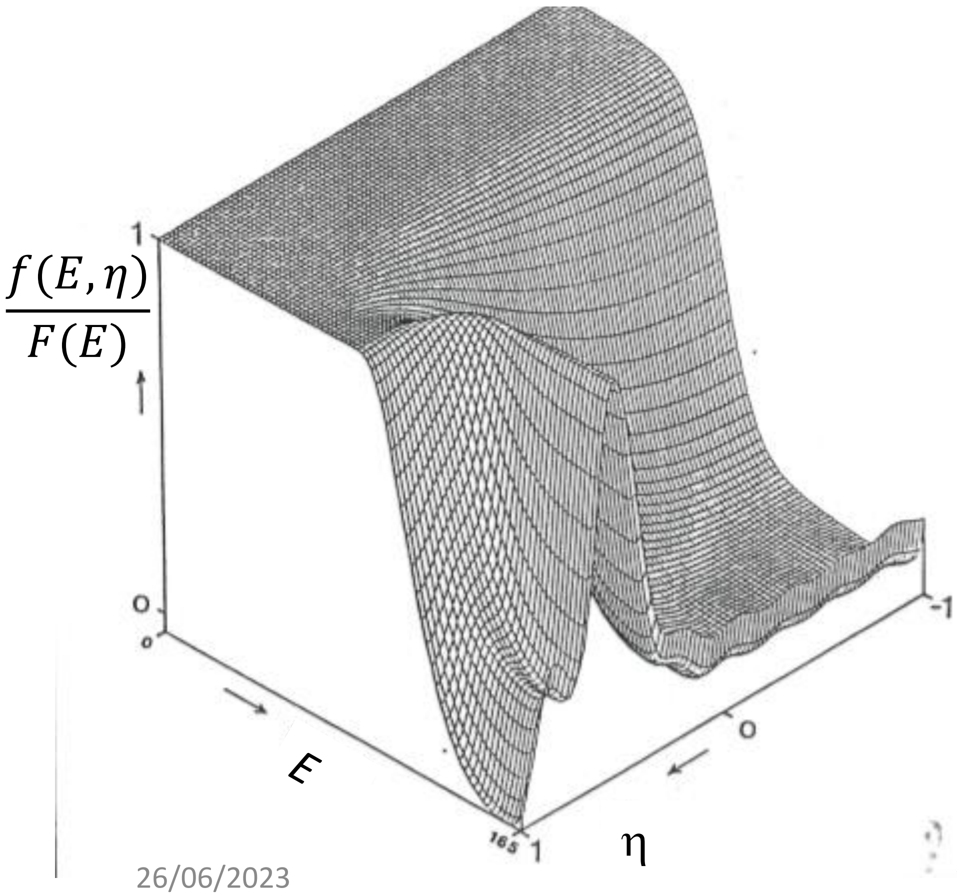


J.D Strachan et al Plasma Phys. Control. Fusion **36** (1994) B3-B15.

M. Keilhacker et al 1999 Nucl. Fusion **39** 209

Neutral beam current drive

- Non-perpendicular NBI obviously give rise to a fast ion current



- To get some insight we consider a source of the type,

$$S = \frac{S_0}{4\pi v_0^2}\delta(v - v_0)\delta(\eta - \eta_0)\delta(\theta - \theta_0)$$

With η_0 close to one and

$$E_{inj} = \frac{1}{2}mv_0^2$$

- We can write the flux surface averaged fast ion current as

$$j_{NBf} = Ze\left\langle \int_0^\infty 2\pi v^2 dv \int_{-1}^{1} v\eta f_0\left(v, \Lambda(\eta,\psi,\theta), P_\varphi(v,\eta,\psi,\theta)\right) d\eta \right\rangle_{Flux\ surface}$$

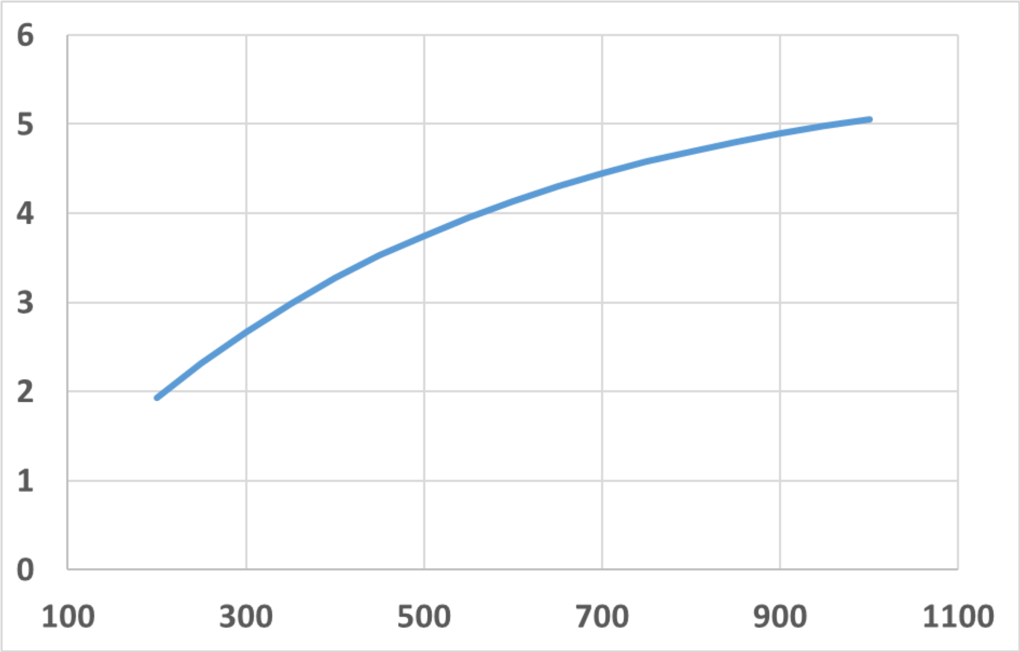
- Neglecting trapped particle effects and FOW effects one finds by expanding f_0 in Legendre polynomials

$$\frac{j_{NBf}}{p_{NBI}} = \frac{Zet_s\eta_0 v_c}{E_{inj}}\left(1+\frac{v_c^3}{v_0^3}\right)^{\frac{v_\gamma^3}{6v_c^3}} \int_0^{\frac{v_0}{v_c}} \left(\frac{x^3}{x^3+1}\right)^{\frac{v_\gamma^3}{6v_c^3}+1} dx$$

- This represents the upper limit of the fast ion current density (trapping effects will reduce it).

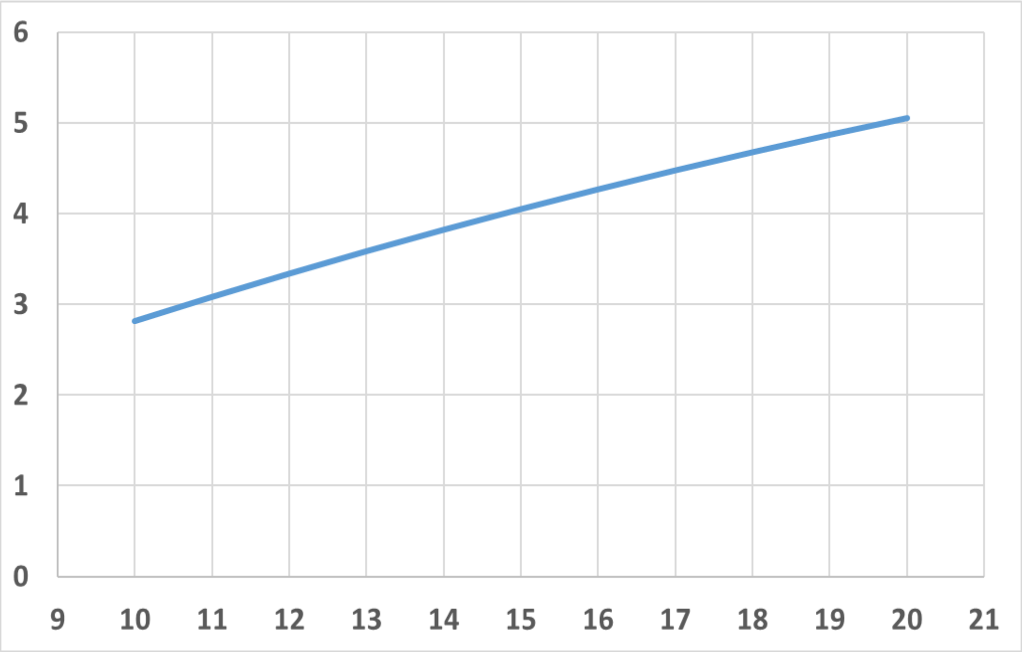
- ITER type plasma, tangential D injection (i.e. $\eta = 1$), $n_e = 1.0 \cdot 10^{20} m^{-3}$

$j_{NBf}/p_{NBI}\ (Am/W)$



$E_{inj}(keV)$

$j_{NBf}/p_{NBI}\ (Am/W)$



$T_e\ (keV)$

Electron back current

- However, the situation is a little more complicated, the fast ions “drags along” electrons, i.e. there is an electron back current

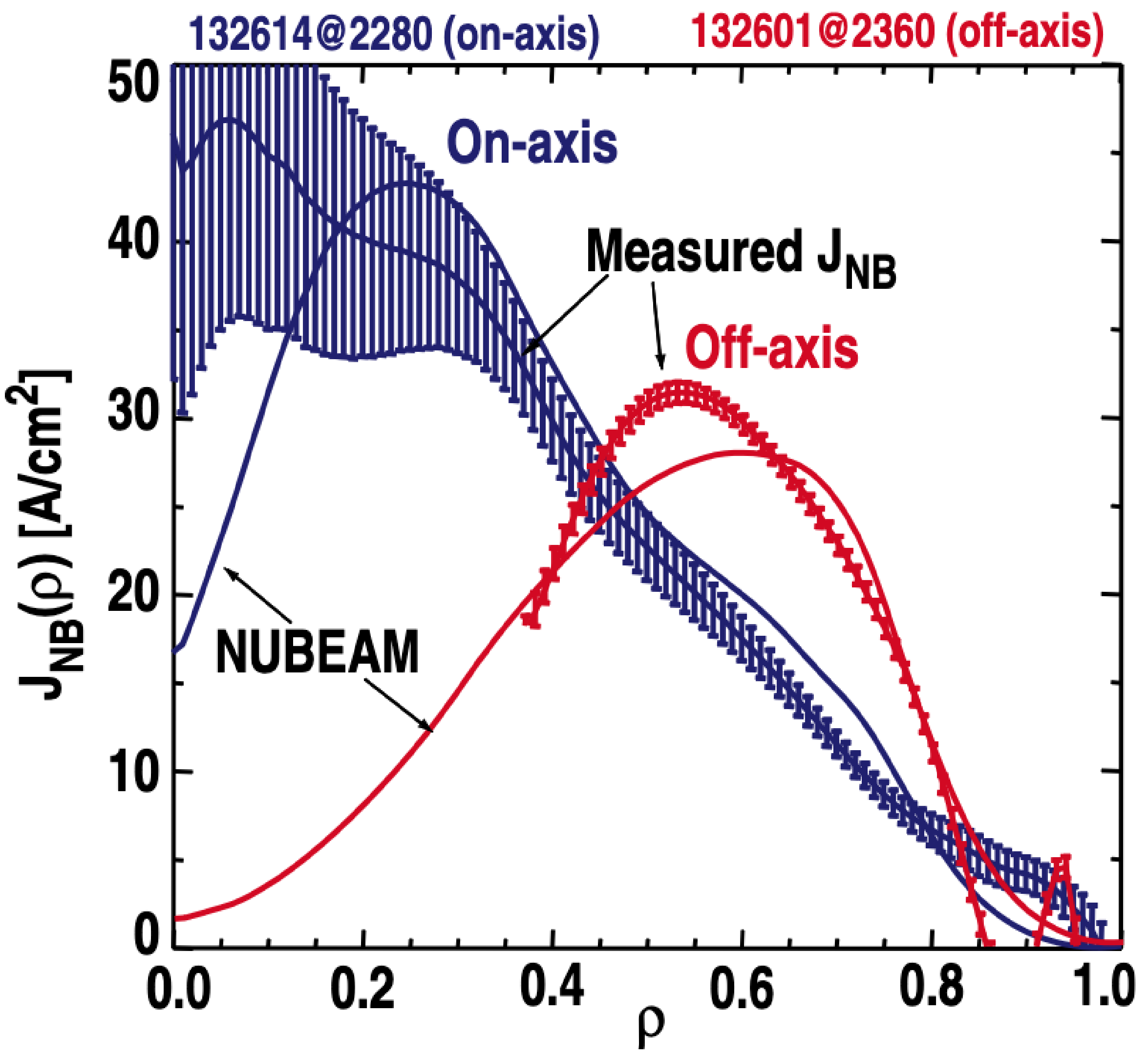
$$j_{NBCD} = j_{NBf}\left[1 - \frac{Z}{Z_{eff}}(1 - G)\right]$$

- To calculate the radially dependent term *G* is complicated*. An approximate expression is $G = 1.46\sqrt{\varepsilon}A(Z_{eff})$; with $A(Z_{eff})$ ~ 1-4 for 1<$Z_{eff} < 4$.
- For Z_{eff} = 2, we can see that the total current is about 50% of the fast ion current in a D or DT plasma.

See e.g M. Honda et al, Nuclear Fusion, **52**, 023021 (2012)

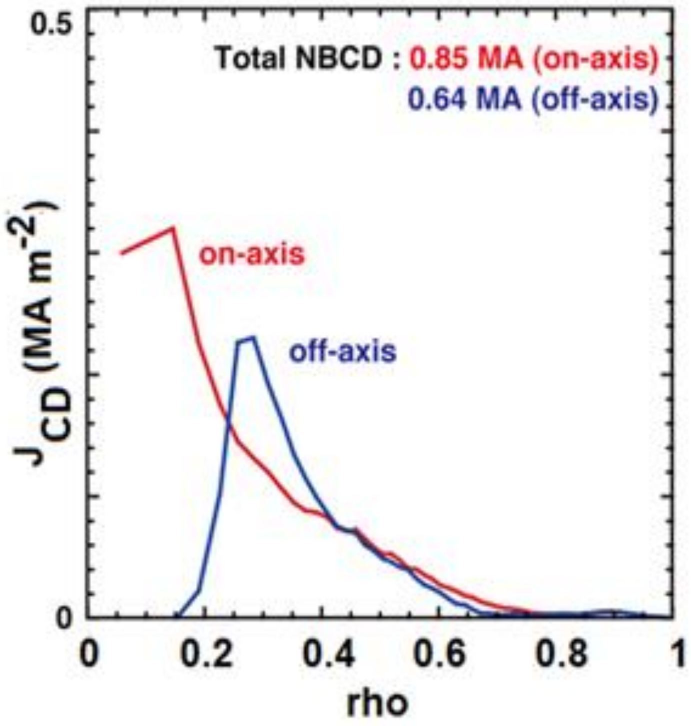
Comparison experiment and simulations

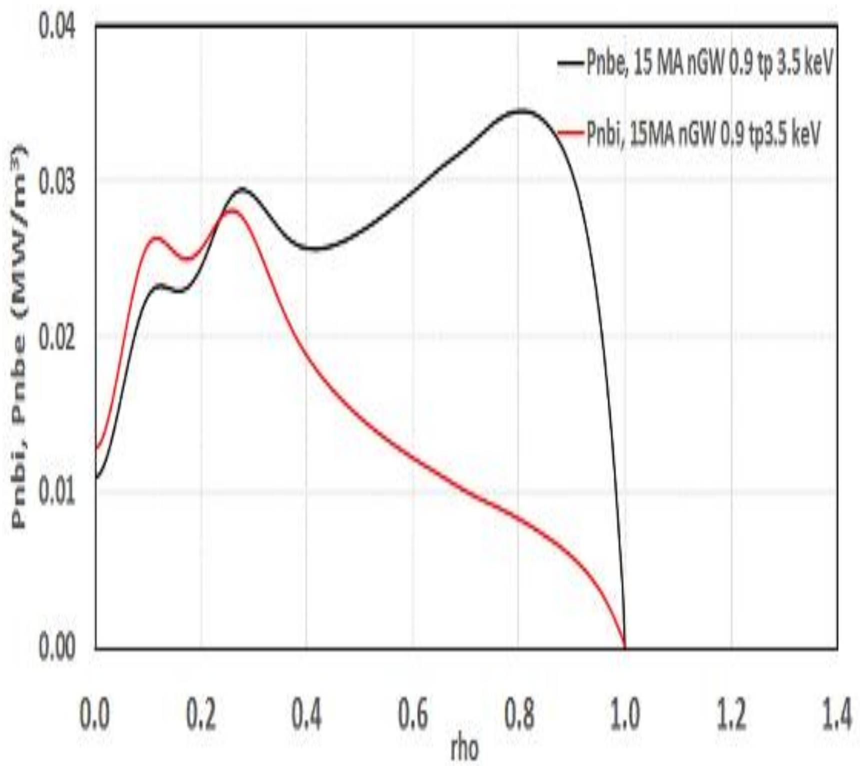
- Experiments on DIII-D with on and of axis NBI.
- Generally good agreement with the orbit following Monte Carlo Code NUBEAM (used in TRANSP).
- Virtually no anomalous effects found D.C. Pace et al. PoP **20**, 056108 (2013)
- In ASDEX-Upgrade the situation is less clearcut and anomalous effects may play a role, see Rittich, D.(2018). https://hdl.handle.net/21.11116/0000-0002-8E89-4



J.M. Park et al Physics of Plasmas **16**, 092508 (2009)

- Simulation for P_{NBI} = 33 MW ITER plasma



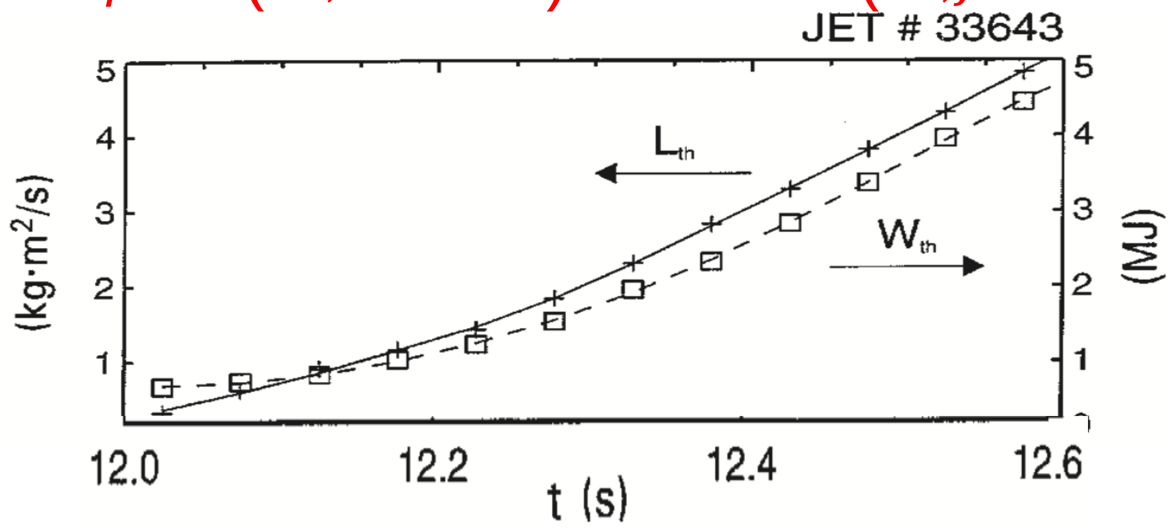


M J Singh et al. New J. Phys.**19** (2017)055004

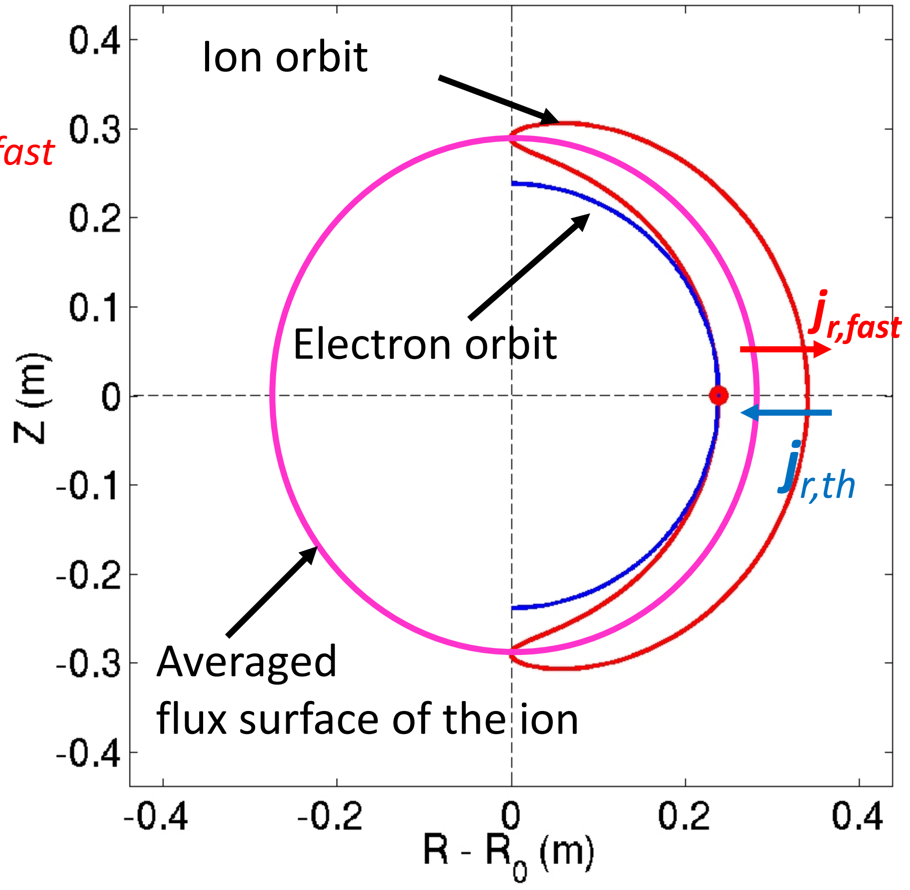
NBI and rotation: momentum transfer

- Passing NBI ions via collisional slowing down/pitch angle scattering.
- NBI ions born on trapped orbits: transfer their momentum during one bounce time
  - Electron and ion orbits have different width →$j_{r,fast}$
  - Quasi neutrality → thermal radial current $\vec{J}_{r,th} = -\vec{J}_{r,fast}$ →

$$T_\varphi = \left(\vec{J}_{r,th} \times \vec{B}\right) \cdot \hat{\varphi} = -\left(\vec{J}_{r,fast} \times \vec{B}\right)$$

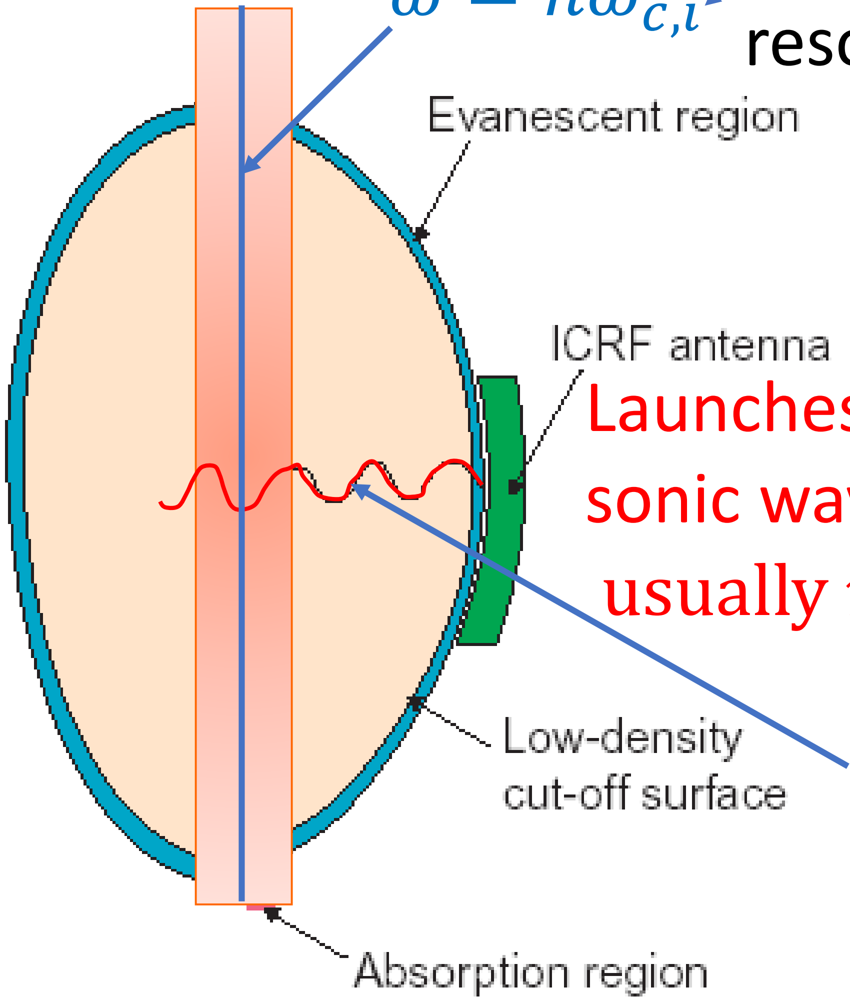


K.-D. Zastrow et al 1998 Nucl. Fusion 38 257



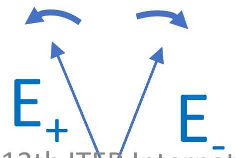
Ion Cyclotron Resonance Frequency (ICRF) heating

$\omega = n\omega_{c,i}$ ← Here i labels a resonant ion species

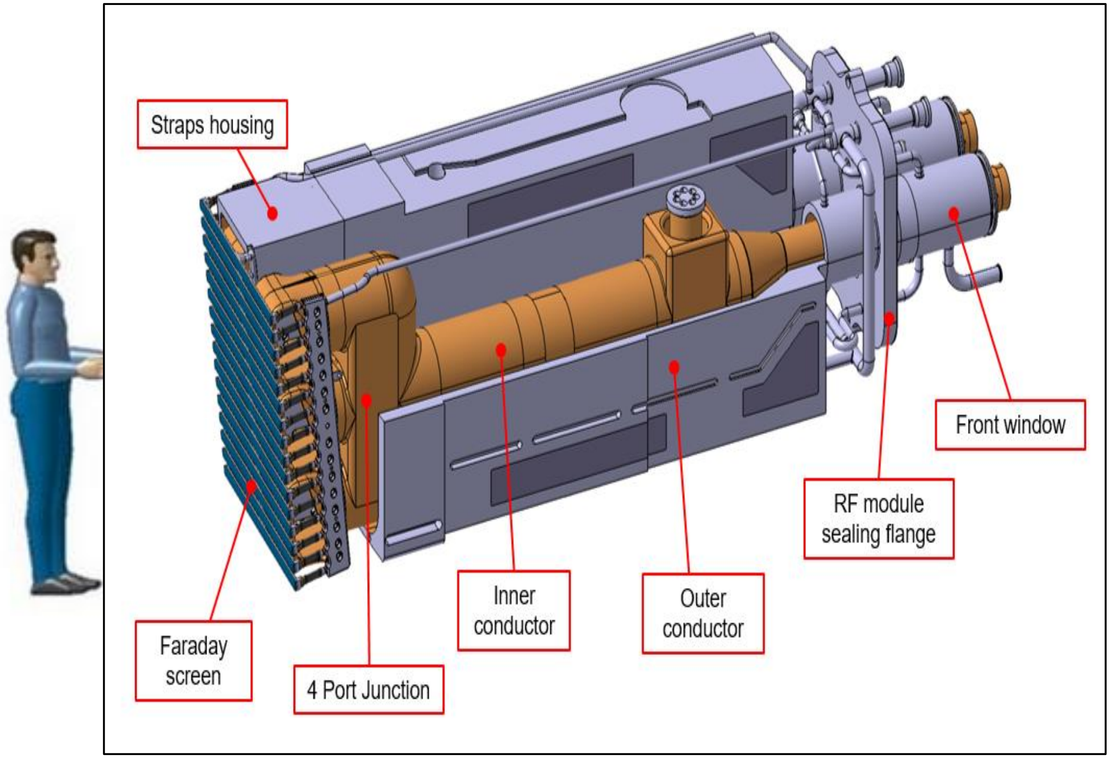


Launches fast magneto-sonic wave, $f \sim n f_{c,i}$, usually $n = 1 - 3$

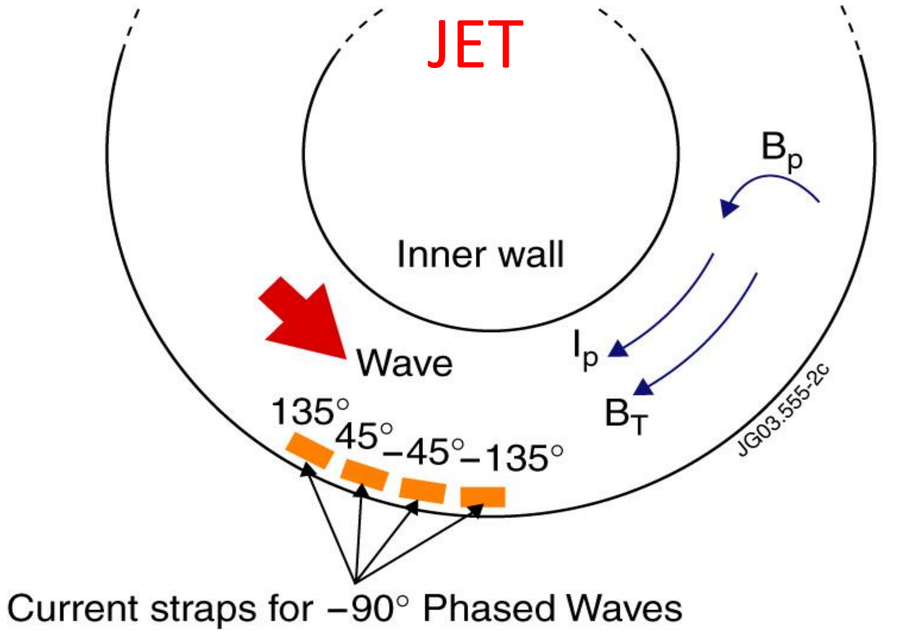
$$\vec{E} = \vec{E}_{+} + \vec{E}_{-}$$

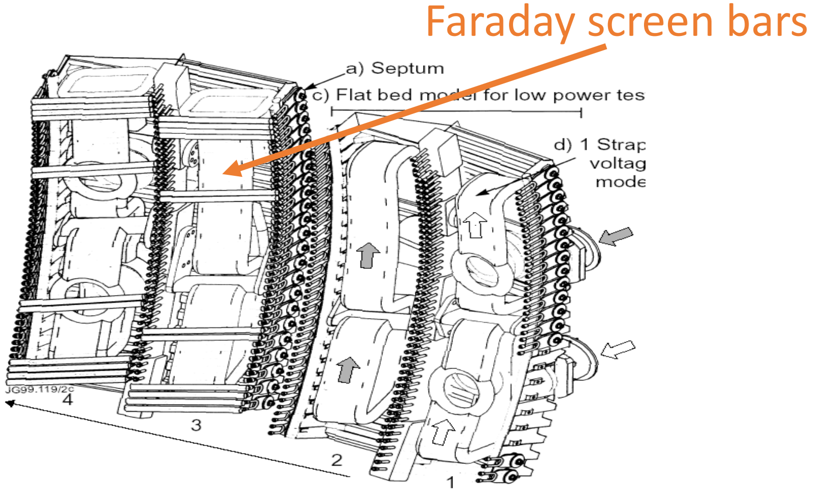


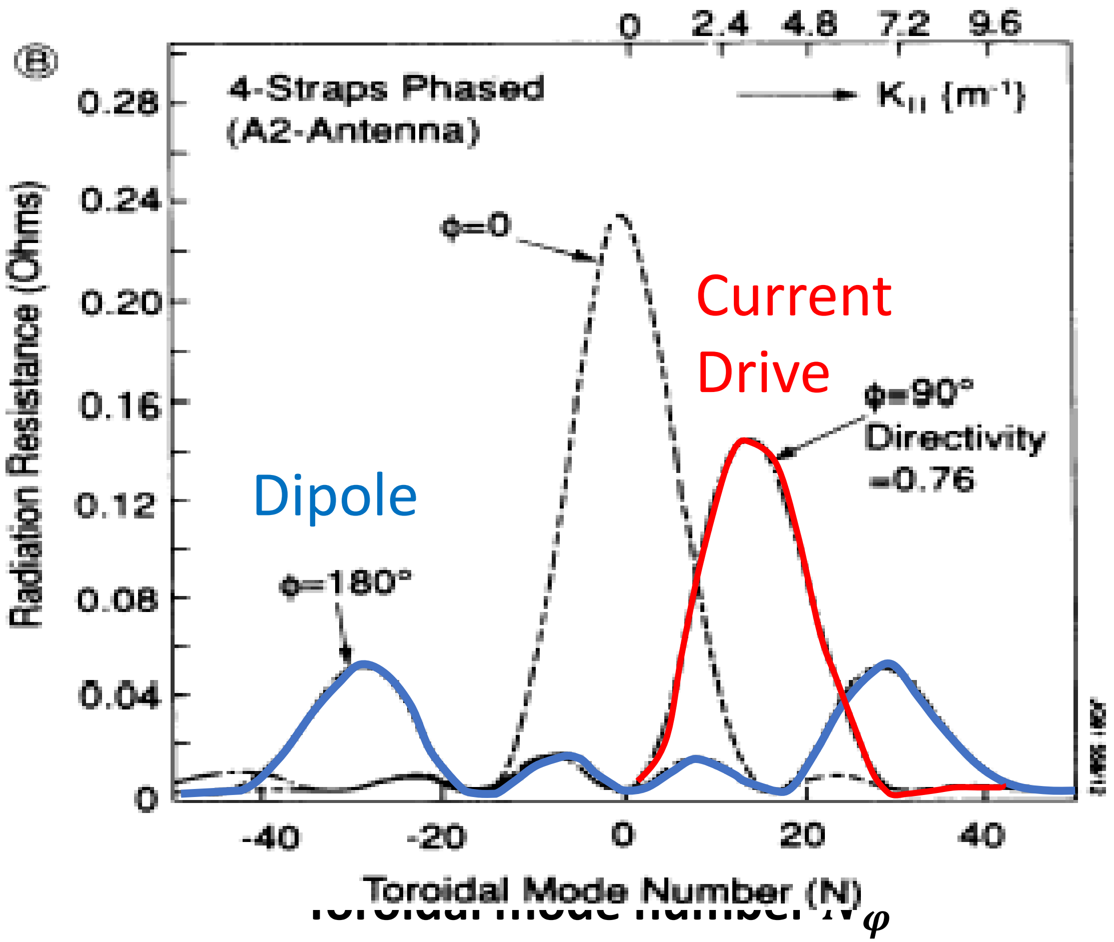
ITER ICRF antenna



ICRF antenna spectra





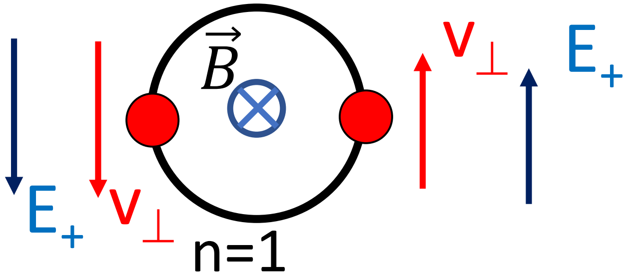


$$\vec{E} = \vec{E} e^{iN_\varphi \varphi}$$

$$k_{||} \approx \frac{N_\varphi}{R} \quad \text{for} \quad N_\varphi \gg 1$$

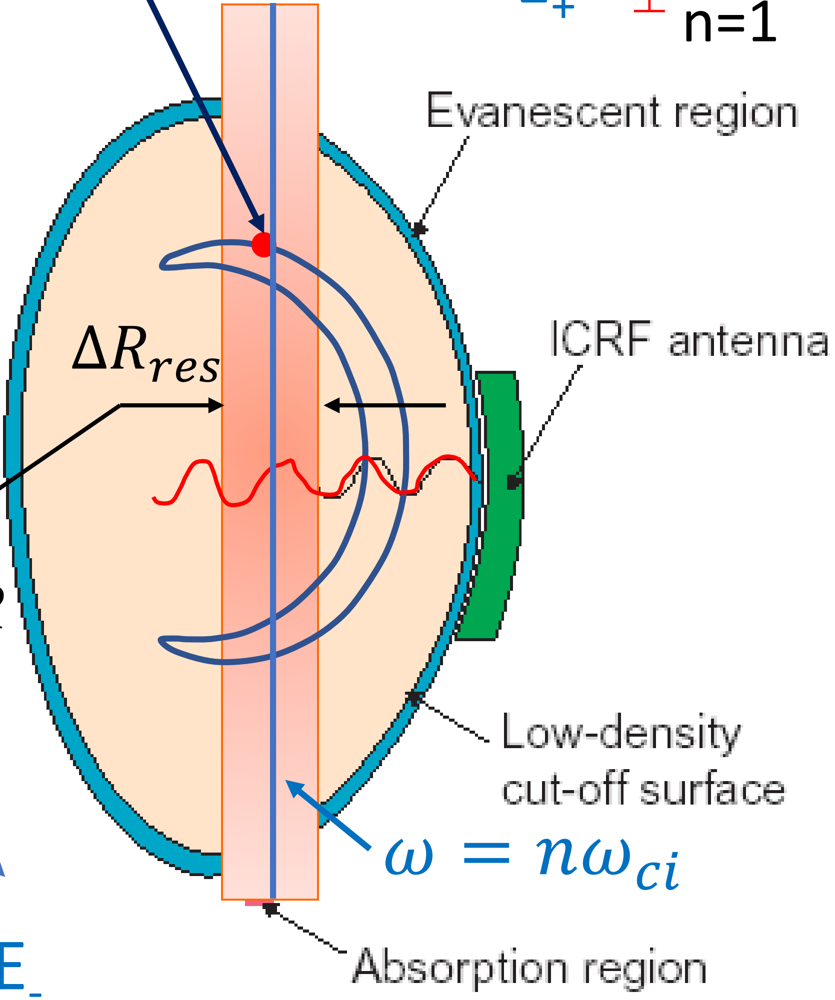
Wave particle interaction

$$\omega - k_{\parallel} v_{\parallel} = n\omega_{ci}$$

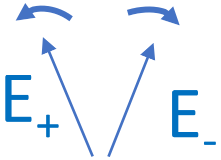


$$\Delta v_{\perp} \sim E_{+} sin(\phi)$$

"Kick" in velocity



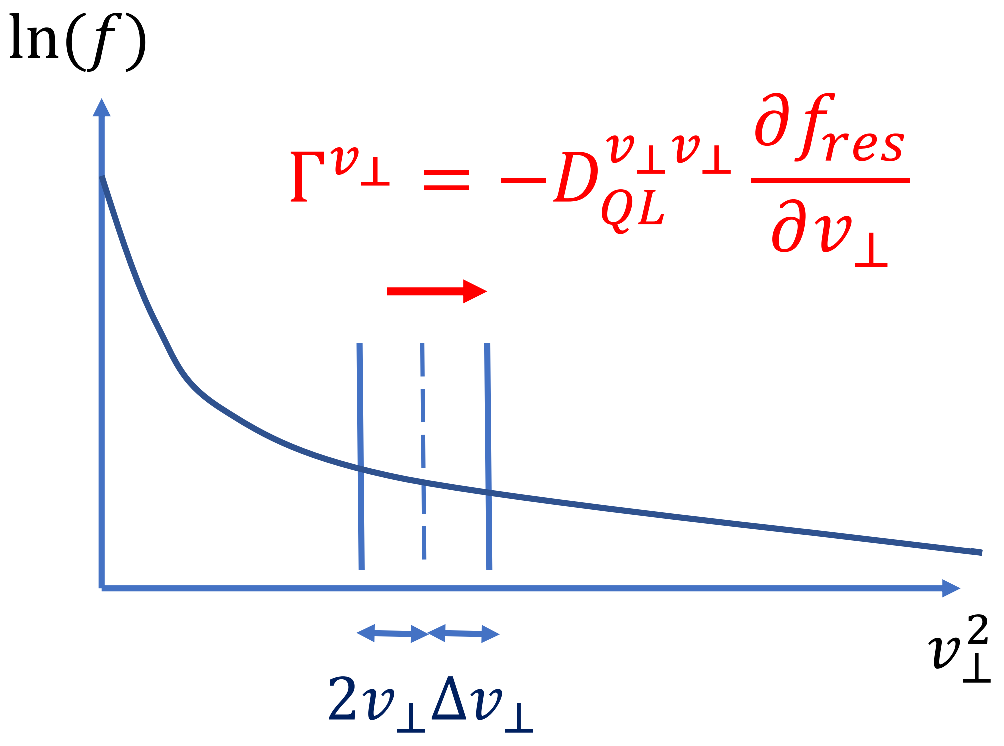
$$\Delta R_{res} \sim \frac{k_{||}\sqrt{\langle v_{||}^{2} \rangle}}{n\omega_{ci}} R$$



- ϕ is the phase between the cyclotron motion and the wave phase;
- E_+ is the left hand polarised component, rotating the same direction of the ions, of the wave electric field.
- We will see later that also the E_- component can give rise to a "kick" in velocity, but it is an FLR effect.

Effective ICRF heating – a random walk process

- Effective ICRF heating requires that ϕ is randomised between successive "kicks"[1,2,3]



- ϕ randomised between successive "kicks" → random walk process characterised by:

$$D_{QL}^{v_\perp v_\perp} \sim \frac{\langle (\Delta v_\perp)^2 \rangle|_\phi}{2\tau_b}$$

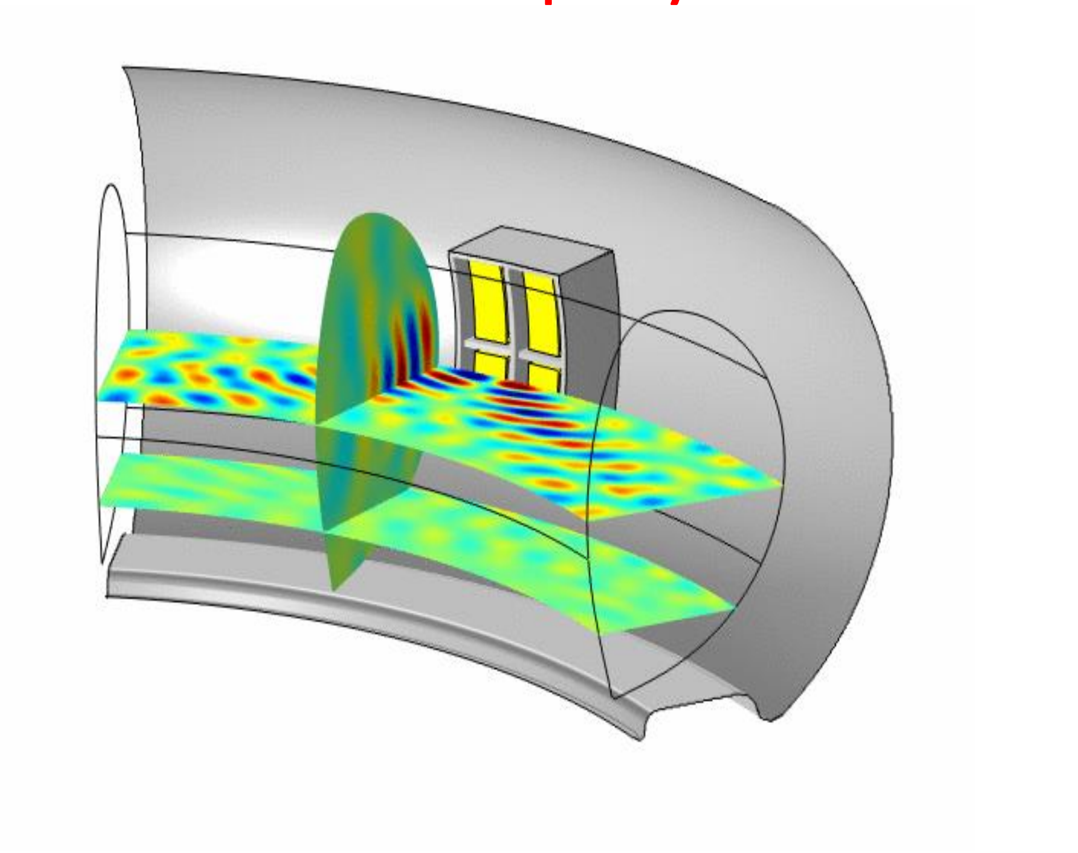
- For n=1 and lowest order in $k_\perp v_\perp/\omega_{ci}$

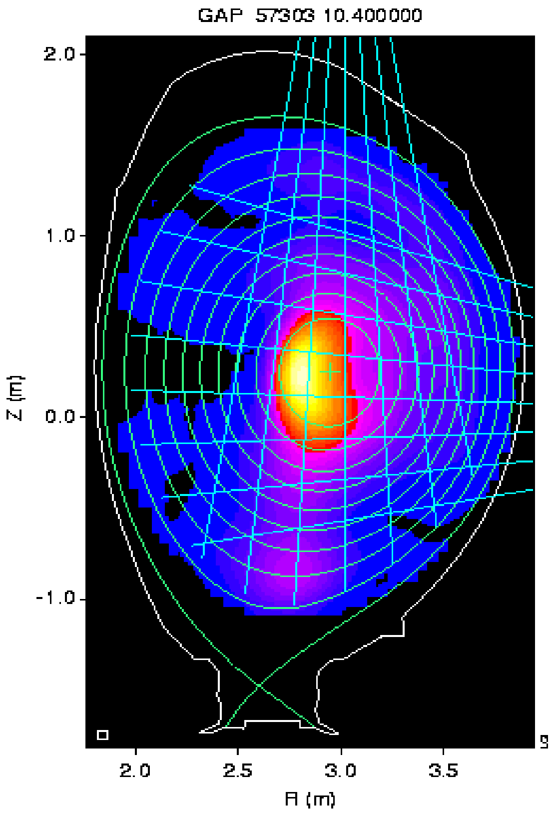
$$D_{QL}^{v_\perp v_\perp} \sim |E_+|^2$$

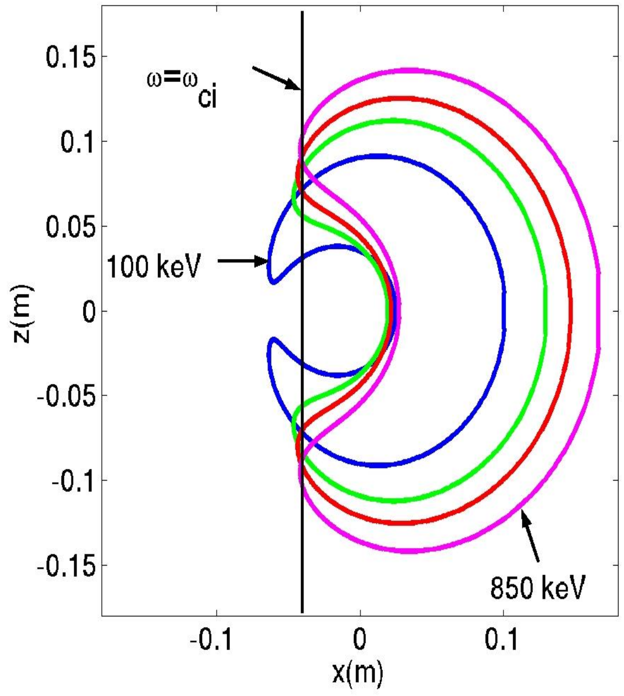
- Thus, ICRF heating is essentially a diffusive process in velocity space.

[1]T.H. Stix "Waves in Plasmas" AIP 1992
[2]P. Helander, M. Lisak Phys. Fluids **B 4** (7), 1927
[3]V. Bergaud et al. Phys. Plasmas, **8**, 2001, 139

ICRF physics modelling in a nutshell







$$\nabla \times \nabla \times \vec{E} = \overleftrightarrow{\varepsilon}\left(f_{0,1}, \cdots, f_{0,n}\right) \cdot \vec{E} + i\omega\mu_0 \vec{J}_{ext}$$

$$\frac{\partial f_{0,n}}{\partial t} = \langle C(f_{0,n}) \rangle + \left\langle Q(f_{0,n}, \vec{E}) \right\rangle$$

ICRF schemes

Cold plasma dispersion for single ion plasma :

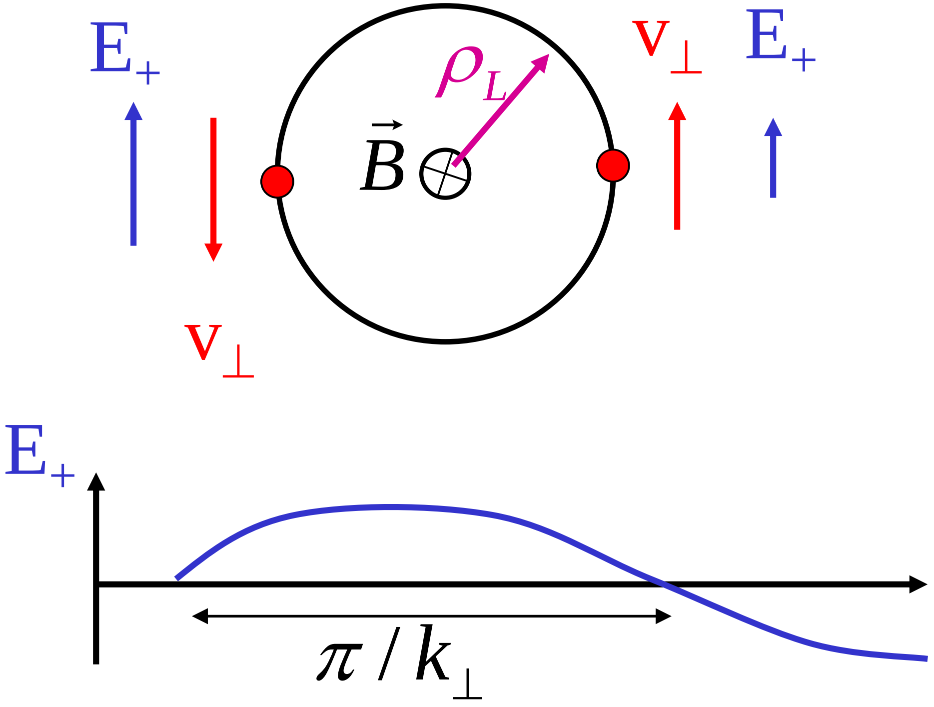
$$\frac{E_+}{E_-} = \frac{\frac{\omega_{ci}}{\omega + \omega_{ci}} - N_\parallel^2}{\frac{\omega_{ci}}{\omega - \omega_{ci}}\omega + N_\parallel^2}$$

$\omega \sim \omega_{ci}$ does not work for a majority ion species

- There are a few options:
  - Minority heating: introduce a minority ion species that is resonant with the waves, which has higher cyclotron frequency than the majority species, (H)D, (^{3}He)D, (D)T, the ion-ion hybrid layer that is introduced ensures a significant E_+ at the $\omega = \omega_{c,min}$, typically $n_{min}/n_{maj} \sim 1 - 10\%$. High power and minority absorption → energetic ions
  - Higher harmonic heating: $\omega = n\omega_{c,maj}, n \geq 2$, the ITER ICRF system is designed for $\omega = 2\omega_{c,T}$ at the full magnetic field. Absorption is an FLR effect → energetic ions
  - Three ion scheme: in a plasma with two majority species introduce a third, minority, species with $\omega = \omega_{c,min}$ where E_+/E is max. Ye. O. Kazakov et al Nature Physics **13**, (2017)

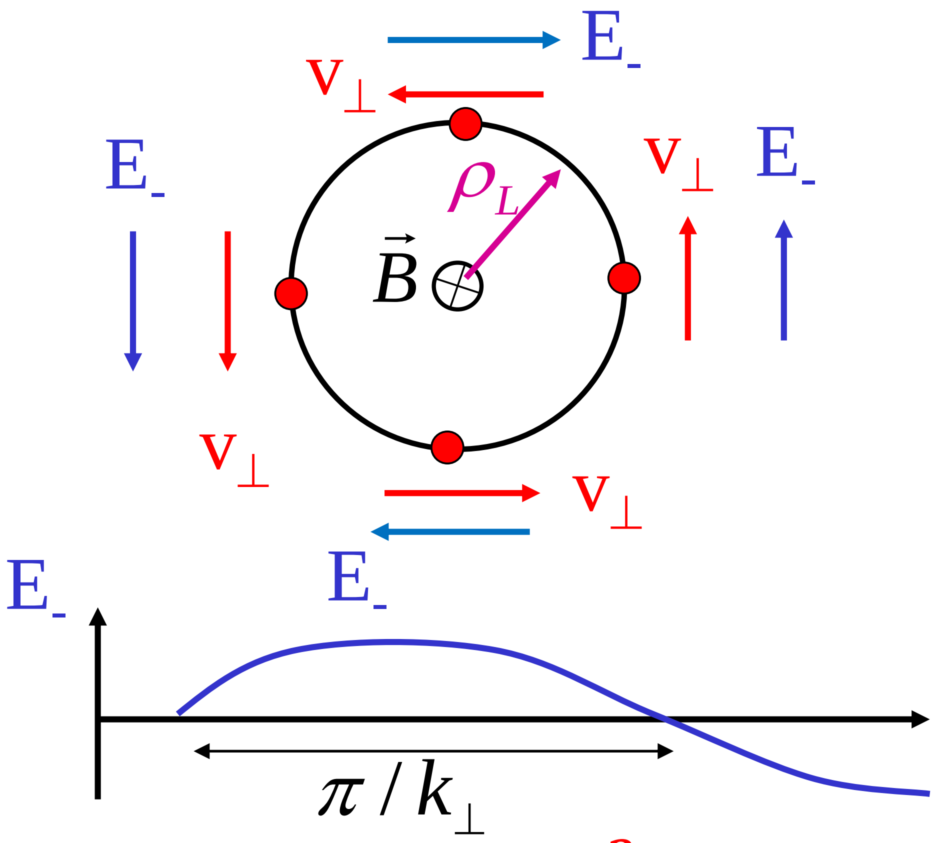
“Kick” mechanisms for $n > 1$ and E_-

$n = 2; \quad E_+$



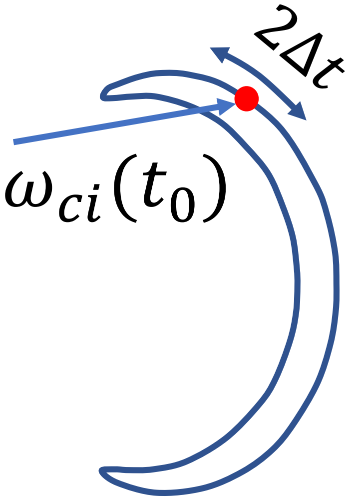
$$\Delta v_\perp \sim E_+ \left(\frac{k_\perp v_\perp}{\omega} \right) sin(\phi)$$

$n = 1; \quad E_-$



$$\Delta v_\perp \sim E_- \left(\frac{k_\perp v_\perp}{\omega} \right)^2 sin(\phi)$$

The kick and stationary phase



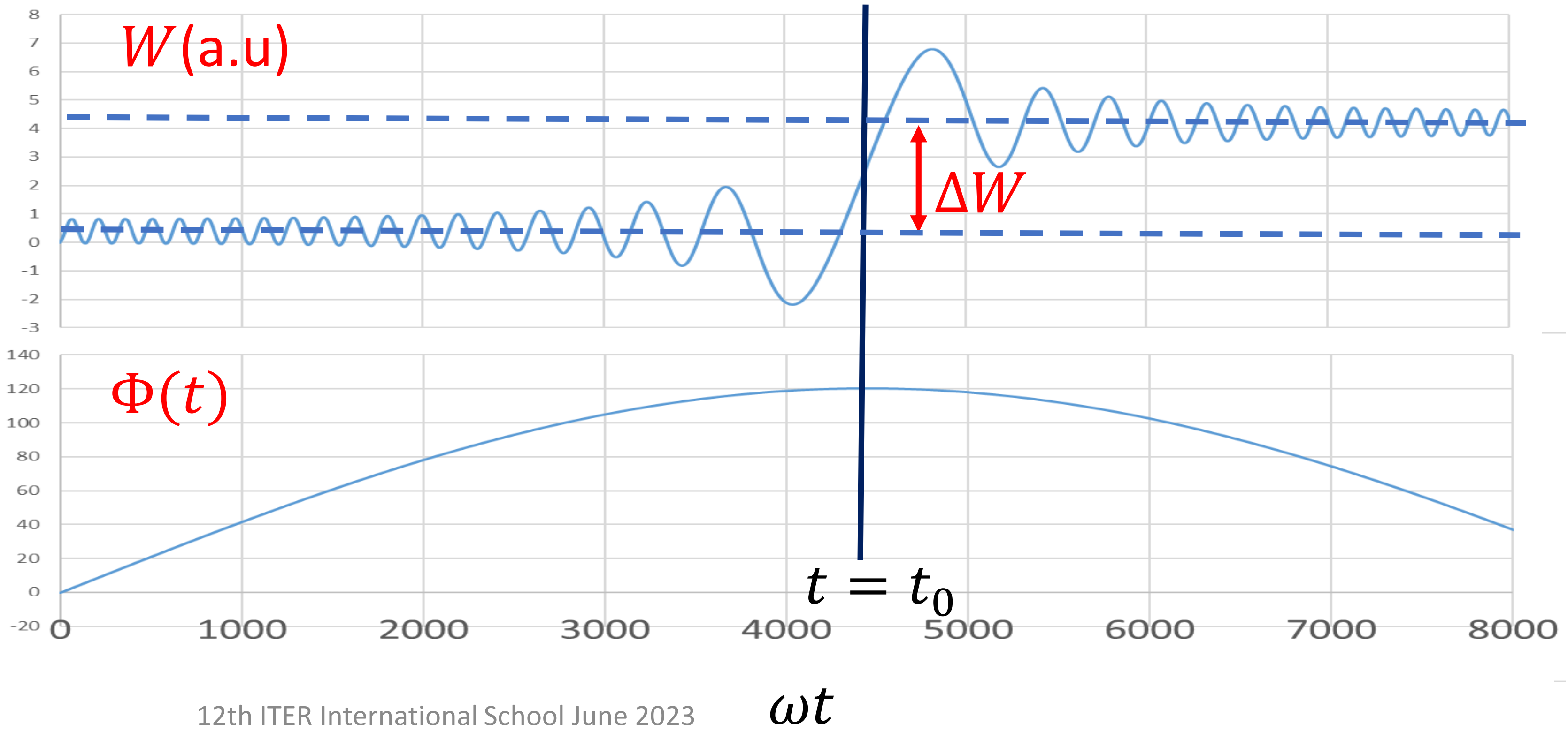
$$\omega - k_\parallel v_\parallel = n\omega_{ci}(t_0)$$

- The change in energy of a particle in given by

ICRF

$$\Delta W = \int_{t-\Delta t}^{t+\Delta t} Ze\vec{E}\cdot\vec{v}dt \approx \int_{t_0-\Delta t}^{t_0+\Delta t} Ze\{E_+ v_\perp cos[\Phi(t)+\phi]+\cdots\}dt$$

$$\frac{d\Phi}{dt} = \left(\omega - k_\parallel v_\parallel - n\omega_{ci}\right)$$



Stationary phase approximation

- The averaged square of the "energy kick" formulated in terms of a random walk diffusion coefficient can be calculated using the stationary phase approximation,

$$D_{N,n,\omega,Res}^{WW} = \frac{\left.\langle(\Delta W)^2\rangle\right|_\phi}{2\tau_b} \approx \frac{\pi(Ze)^2 v_{\perp R}^2}{2\tau_b|n\dot{\omega}_{cR}|}\left|E_+ J_{n-1}\left(\frac{k_\perp v_{\perp R}}{\omega_{cR}}\right) + E_- J_{n+1}\left(\frac{k_\perp v_{\perp R}}{\omega_{cR}}\right)\right|^2$$

- FLR effects are represented by the Bessel functions, and we will see later that they can be very important.

Relations for wave particle interaction via quantum mech.

CHALMERS

$$\Delta W = \hbar\omega \qquad m\Delta v_{||} = k_{||}\hbar \qquad \Delta P_\varphi = Rk_\varphi\hbar = N_\varphi\hbar$$

Resonance condition

$$\Delta W_{||} = mv_{||}\Delta v_{||} = \frac{k_{||}v_{||}}{\omega}\Delta W$$

$$\Delta W_\perp = \left(1 - \frac{k_{||}v_{||}}{\omega}\right)\Delta W = \frac{n\omega_c}{\omega}\Delta W$$

$$\Delta\Lambda = \Delta\frac{W_\perp B_0}{WB} = \left(\frac{\Delta W_\perp}{W} - \frac{W_\perp}{W^2}\Delta W\right)\frac{B_0}{B} = \left(\frac{n\omega_c}{\omega W} - \frac{W_\perp}{W^2}\right)\frac{B_0}{B}\Delta W$$

$$\Delta\Lambda = \frac{n\omega_{c0} - \Lambda\omega}{\omega W}\Delta W$$

$$\Delta P_\varphi = \frac{N_\varphi}{\omega}\Delta W$$

The orbit averaged Quasi-linear operator $\langle Q(f_0)\rangle$

CHALMERS

- Let's denote $\vec{I} = (W, \Lambda, P_\varphi)$.
- We assume that the the energy kicks between resonances are randomised → random walk diffusion coefficient,

$$D_{QL}^{ij} = \sum_{n,N,\omega} \sum_{Res.} \frac{\langle \Delta I^i \Delta I^j \rangle\big|_\phi}{2\tau_b} = \sum_{n,N,\omega} \sum_{Res} D_{n,N,\omega,Res.}^{WW} \frac{\partial I^i}{\partial W} \frac{\partial I^i}{\partial W};$$

$$\frac{\partial \Lambda}{\partial W} = \frac{n\omega_{c0} - \Lambda\omega}{\omega W};$$

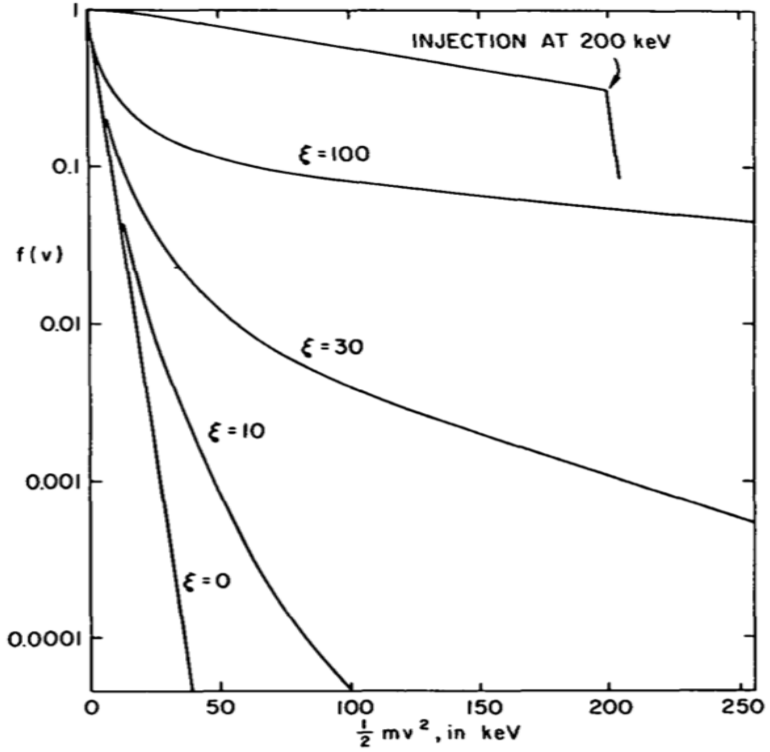
$$\frac{\partial P_\varphi}{\partial W} = \frac{N_\varphi}{\omega}$$

- $\langle Q(f_0)\rangle$ for the wave particle interaction then takes the form:

$$\langle Q(f_0)\rangle = \frac{1}{\sqrt{g}} \frac{\partial}{\partial I^i} \left(\sqrt{g} D_{QL}^{ij} \frac{\partial f_0}{\partial I^j} \right)$$

Properties of ICRF heated distributions

- The collisions are much stronger at low energies than at high → development of a non-Maxwellian tail on the distribution.
- $\Delta\Lambda = \frac{n\omega_{c0} - \Lambda\omega}{\omega W}\Delta W$ → $\Lambda \xrightarrow[\Delta W>0]{} \frac{n\omega_{c0}}{\omega}$; ion with t.p. at resonance, has $\Lambda = \frac{n\omega_{c0}}{\omega}$



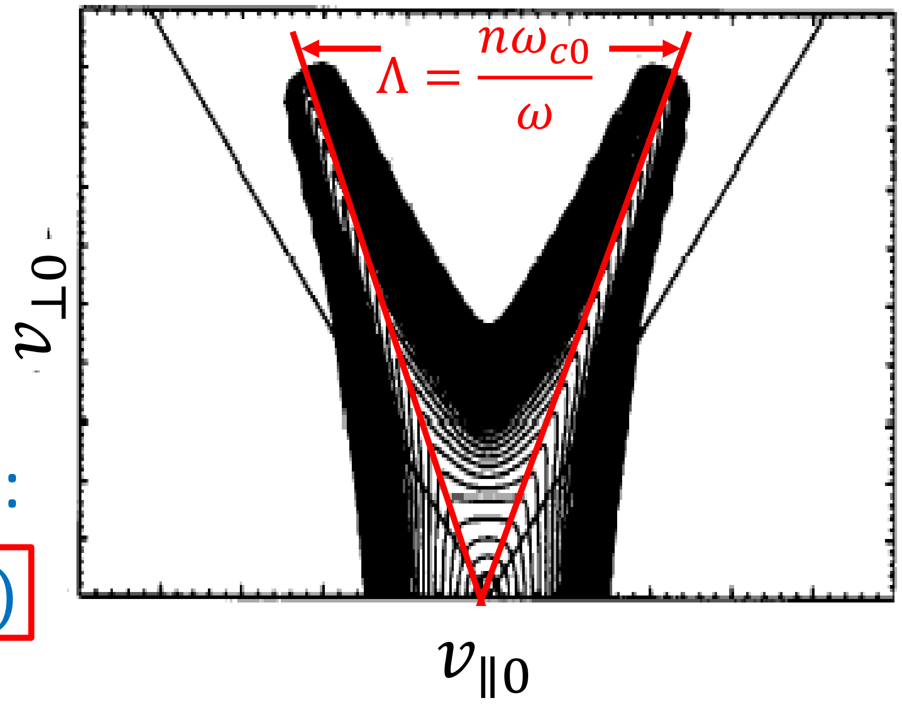
. T.H Stix Nuclear Fusion **15**, 737 (1975)

Minority heating

$$\xi k T_e = \frac{p_{ICRF} t_s}{3nm}$$

Stix tail temperature:

$$kT_{tail} \approx kT_e(1+\xi)$$



G.D. Kerbel and M.G McCoy, Phys. Fluids **28** (1985) 3629.

Approximate scaling of fast ion energy content

- During high power minority heating a large fraction of the absorbing ions are in the tail of the distribution.
- Retaining only slowing down on electrons, neglecting orbit width →

$$0 = \int_r^{r+\Delta r} \frac{1}{2} m v^2 \left[\langle Q(f_0) \rangle + \frac{1}{\sqrt{g}} \frac{\partial}{\partial v} \left(\sqrt{g} \frac{v}{t_s} f_0 \right) \right] \sqrt{g} d^3 I \quad \Rightarrow \quad p_{ICRF} - \frac{2 w_{fast}}{t_s} = 0$$

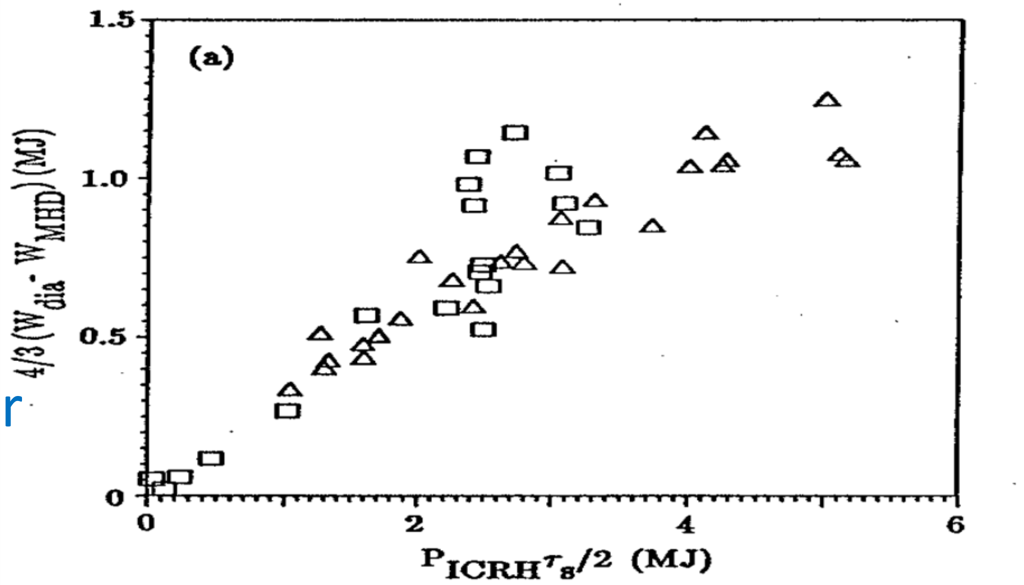
$$w_{fast} \approx \frac{p_{ICRF} t_s}{2}$$

$$t_s \sim \frac{A}{Z^2}$$

$$T_{tail,He-3} < T_{tail,H}$$

$$kT_{tail} \approx \frac{p_{ICRF} t_s}{3nm}$$

Stix tail temperature for $T_{tail} >> T_e$

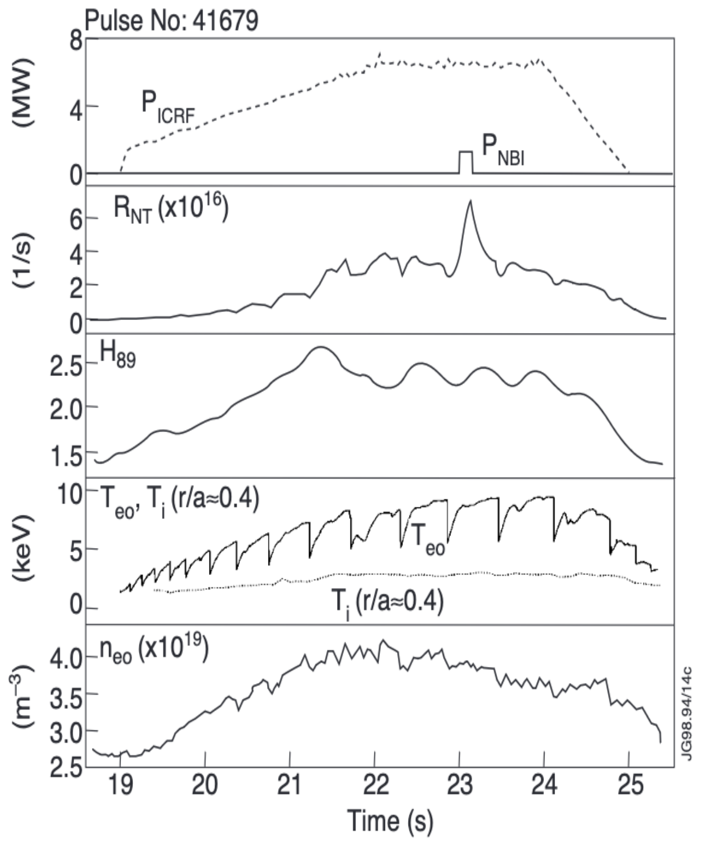


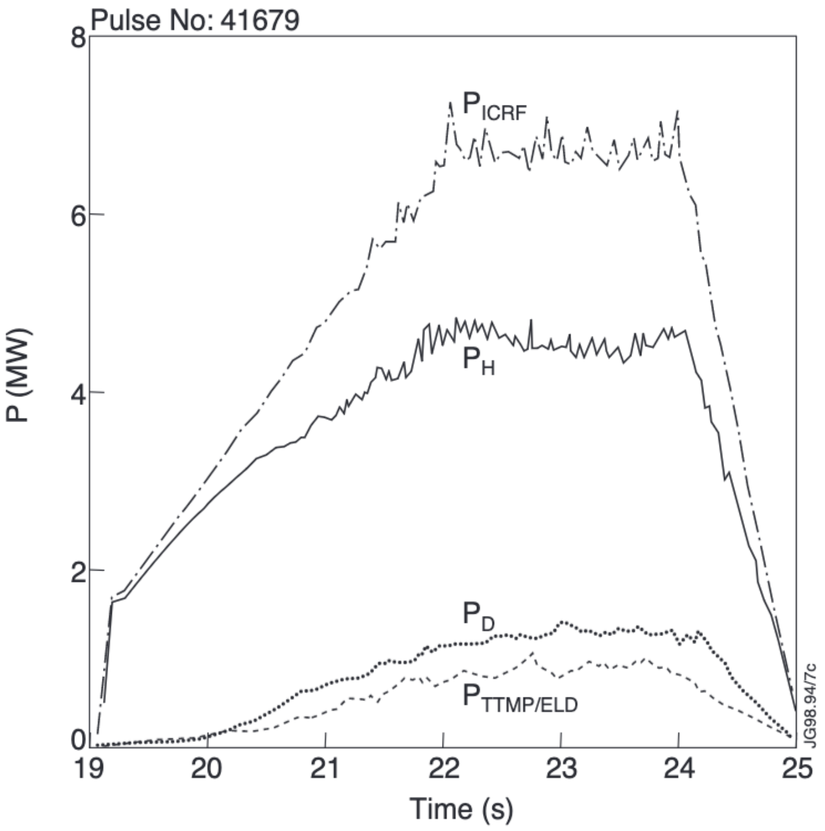
T.H Stix Nuclear Fusion **15**, 737 (1975

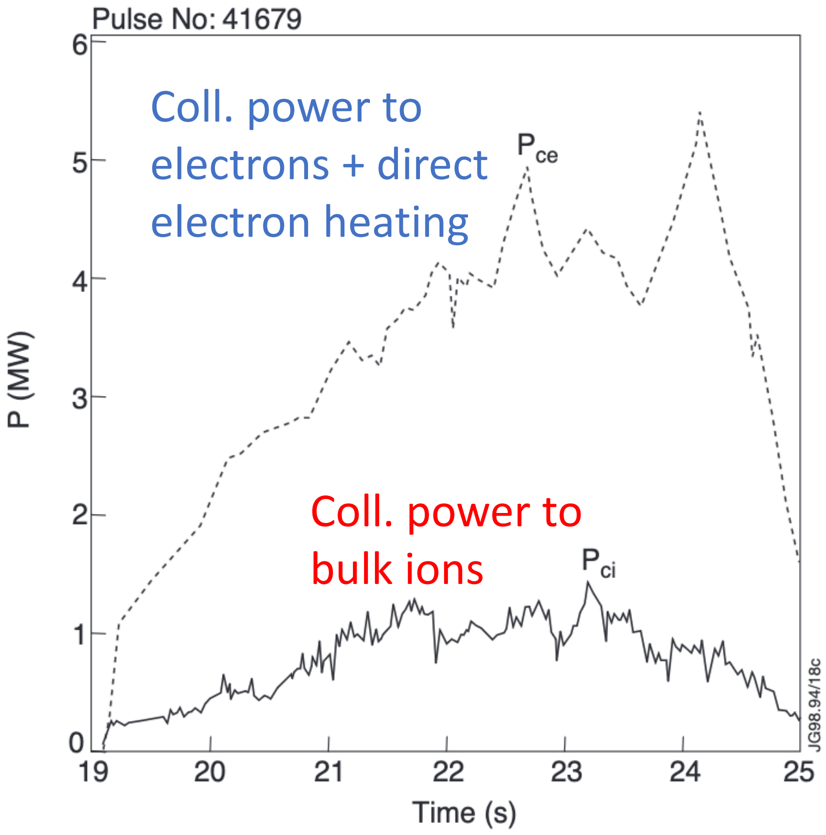
JET team, presented by P.R Thomas (Proc. 12th Int. Conf. Nice, 1988), Vol. 1, IAEA, Vienna (1989) 247.

ICRF often heats bulk electrons more than ions

- PION (simplified self consistent wave prop. and FP code) simulation of H minority ICRF in JET DT plasma with $n_H/(n_D + n_T + n_H) \approx 3\%$ →
- Simulation of neutron rate agreed well with experiment.



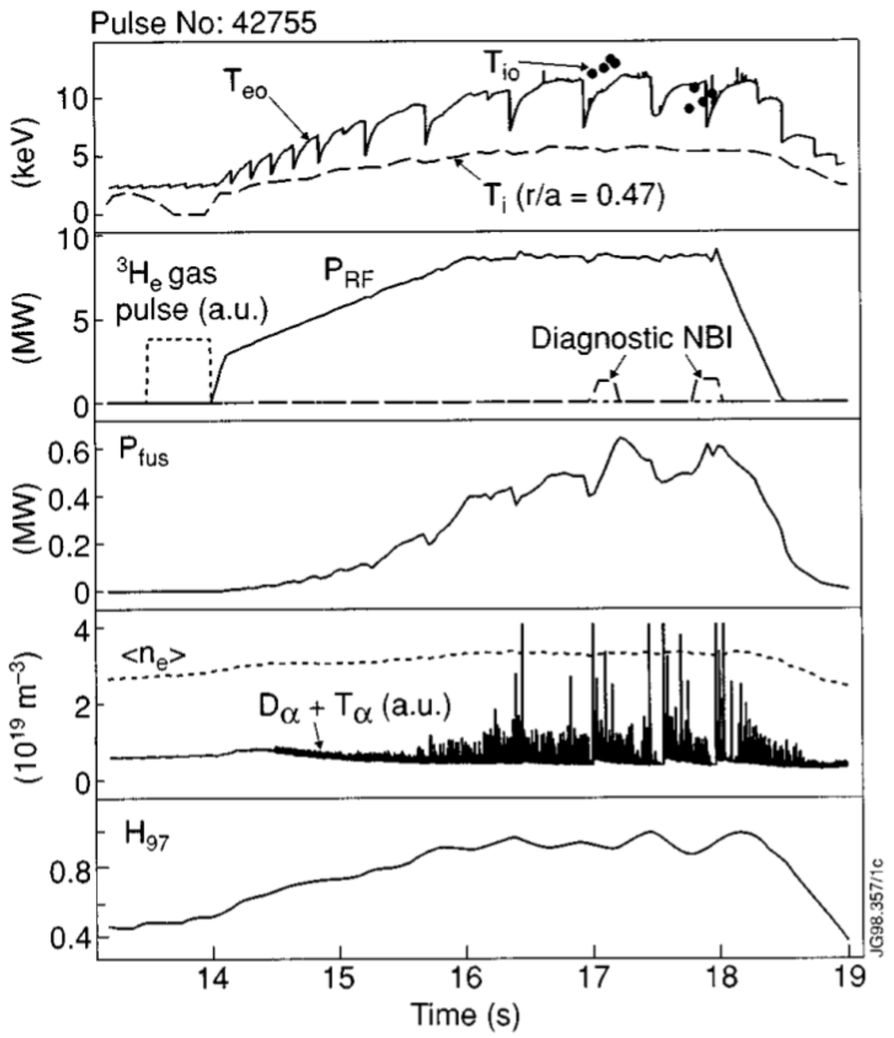


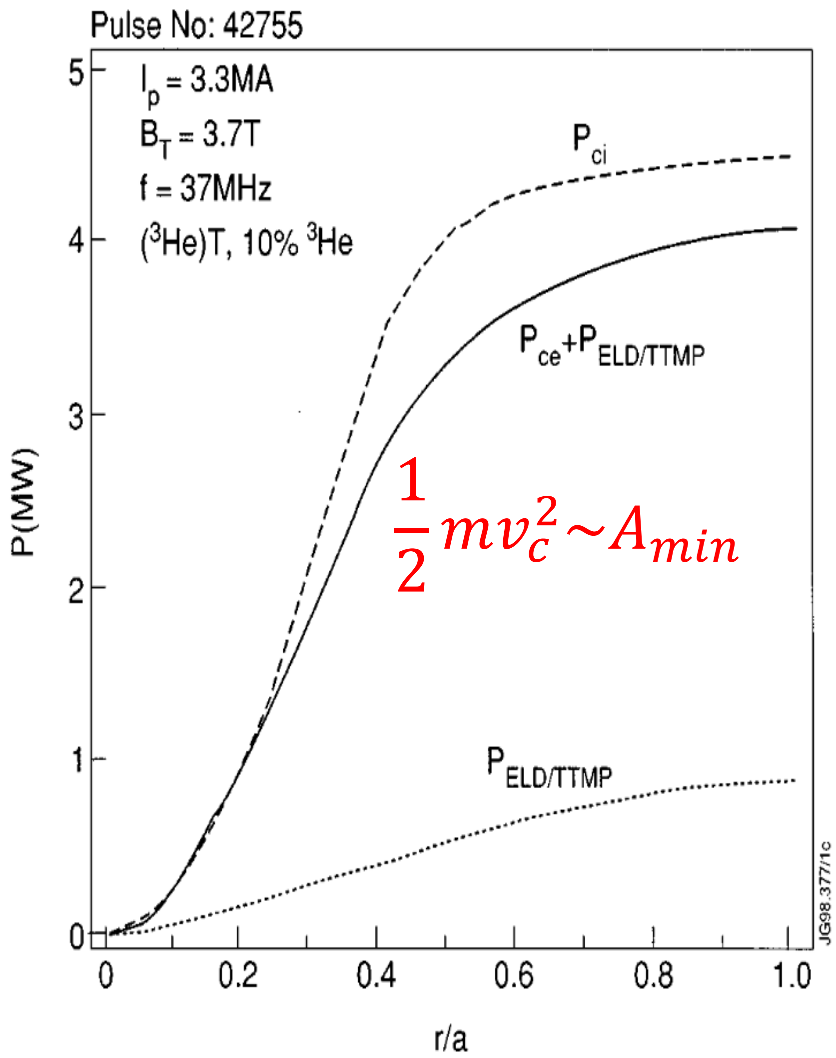


L.-G. Eriksson M. Mantsinen *et al* 1999 *Nucl. Fusion* **39** 337

Dominant bulk ion heating with ICRF is possible

- (^{3}He)DT can produce dominant bulk ion heating; example from JET 1997[1]
- The three ion scheme with absorption on an heavy impurity is also viable[2]





[1]D.F.H. Start et al 1999 Nucl. Fusion **39** 321; V. Bergeaud et al 2000 Nucl. Fusion **40** 35
[2]Ye. O. Kazakov et al AIP Conference Proceedings 1689, 030008 (2015)

The distribution and ICRF power deposition

1) Doppler broadening of the resonance:

- $\Delta R_{res} \sim R\, k_{||} \sqrt{2kT_{\|,tail}/m}/\omega$
- Stix[1] suggested for $T_{\perp,tail} \gg T_{\|,tail}$:

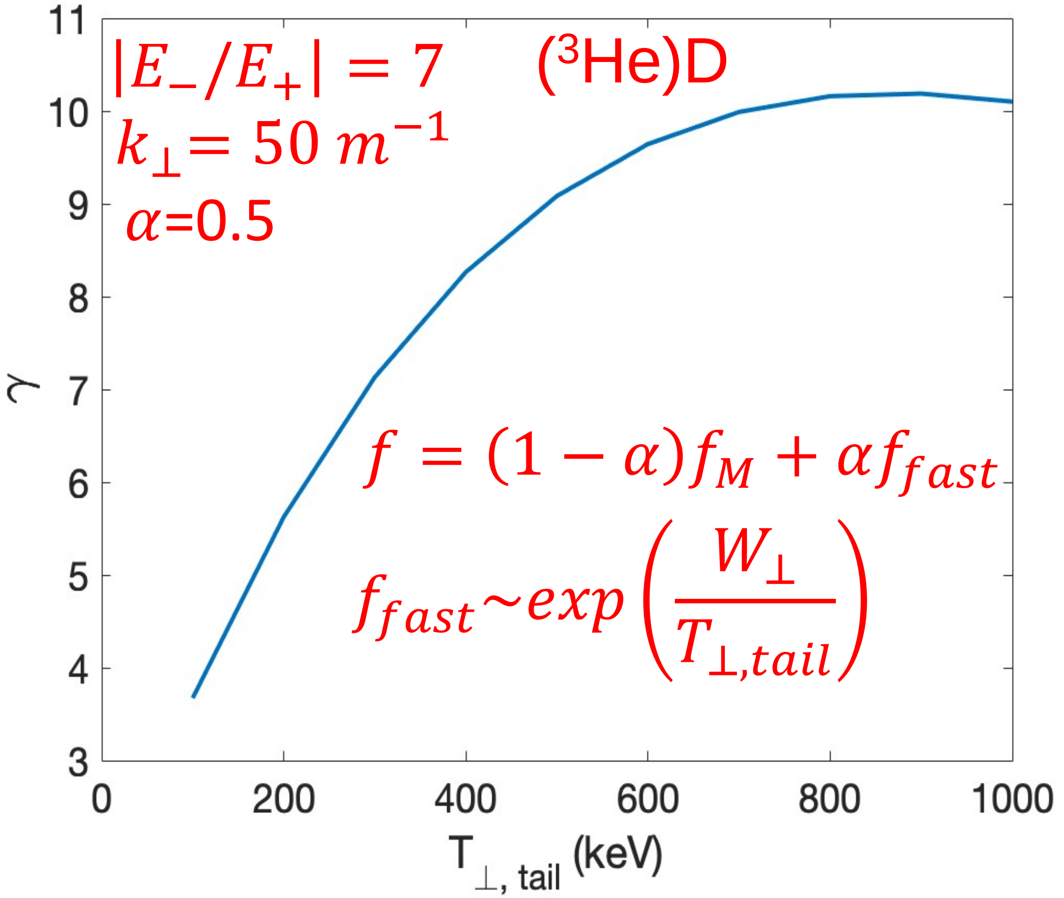
$$T_{\|,tail} \sim m v_\gamma^2/8$$

Ex. (^{3}He)D, $f = 30\ MHz, N_\varphi = 25,$ $T_e = 6\ keV, Z_{eff} = 1.5, \rightarrow$ $T_{\|,tail} = 60\ keV; \rightarrow \Delta R_{res}$ ~ 0.3 m

2) FLR effects.

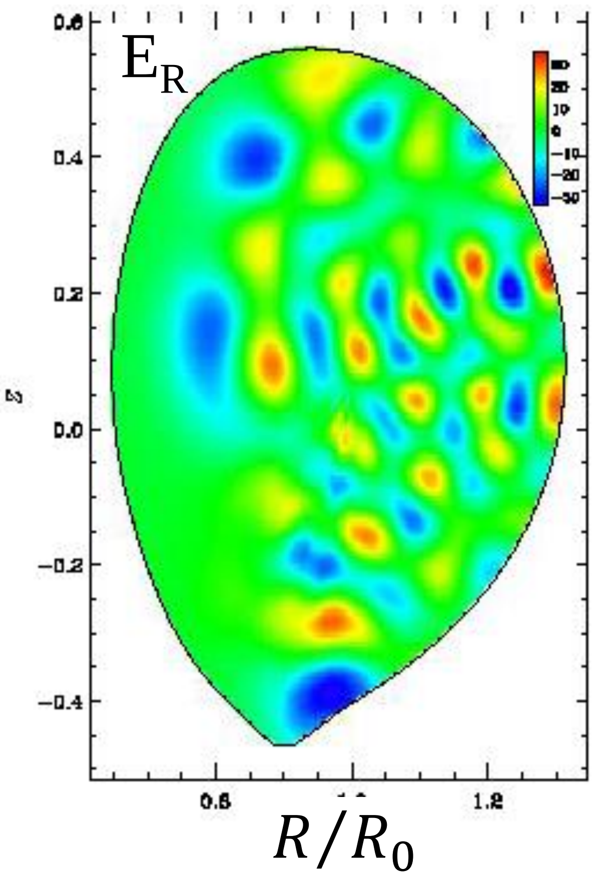
- Wave propagation absorption strength must be equal to that of Fokker-Planck;
- Enhancement over thermal absorption

$$\gamma = \frac{p(f)}{p(f_M)} = \frac{\int m v_\perp^2 D_{QL}^{v_\perp v_\perp} \frac{\partial f}{\partial v_\perp} dv_\perp}{\int m v_\perp^2 D_{QL}^{v_\perp v_\perp} \frac{\partial f_M}{\partial v_\perp} dv_\perp}$$

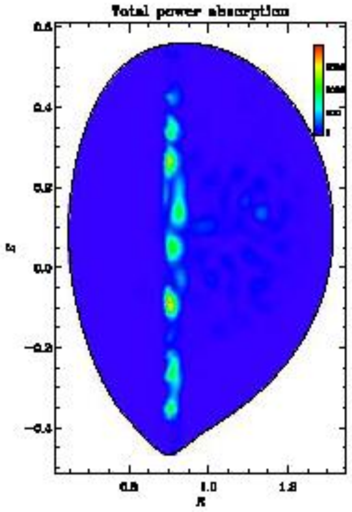


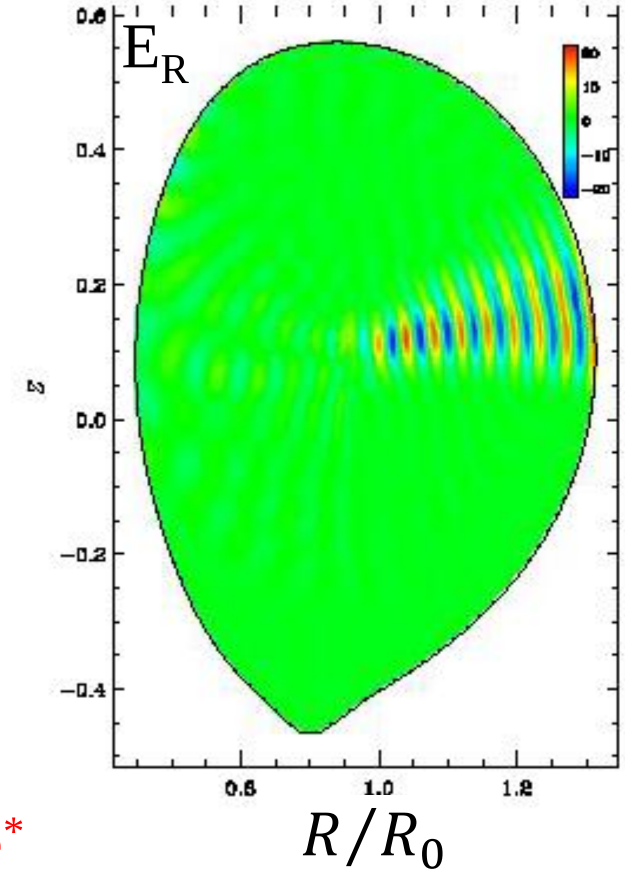
[1]T.H Stix Nuclear Fusion **15**, 737 (1975

ICRF Wave Field strong vs weak damping

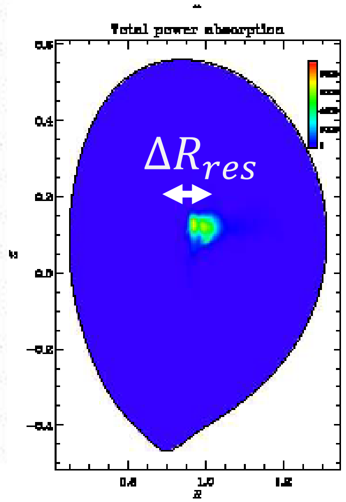


JET, (^{3}He)D
n_{He}/n_D~5%.
$T_{\|He}$= 5keV
N_ϕ=25





JET, (H)D
n_H/n_D~5%.
$T_{\|H}$= 20keV
N_ϕ=25



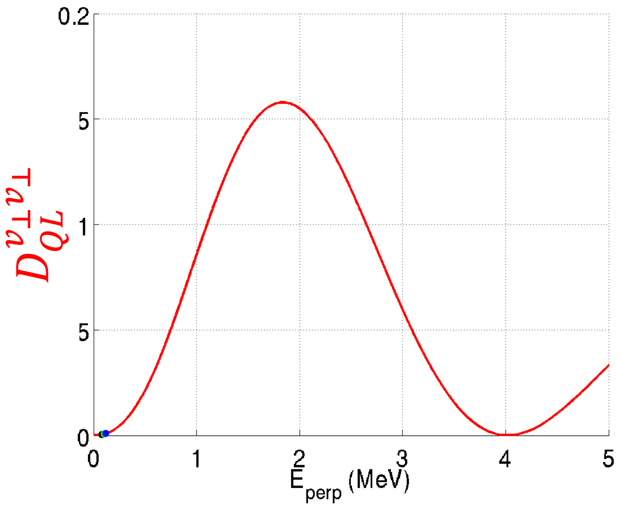
LION code*

Weak damping, the wave field fills much of the cavity

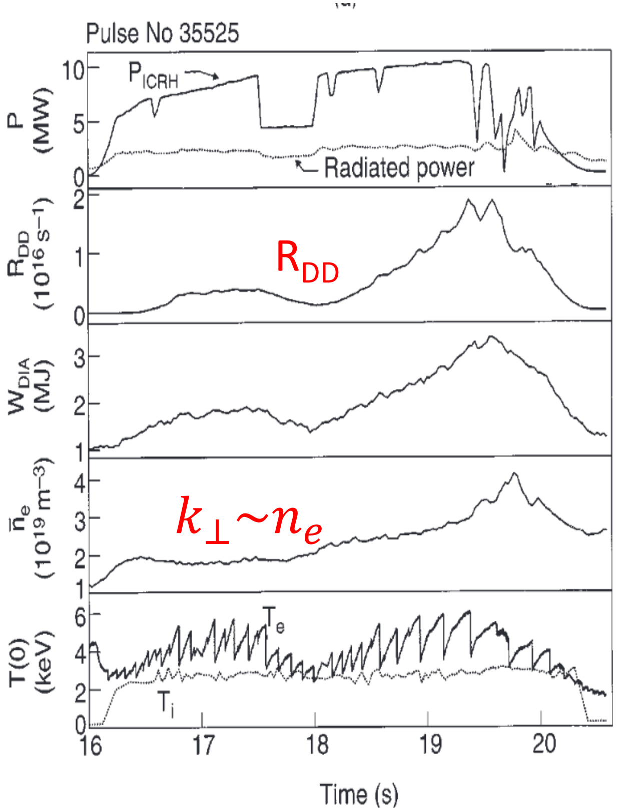
Strong damping, focussing of the wave at first passage.

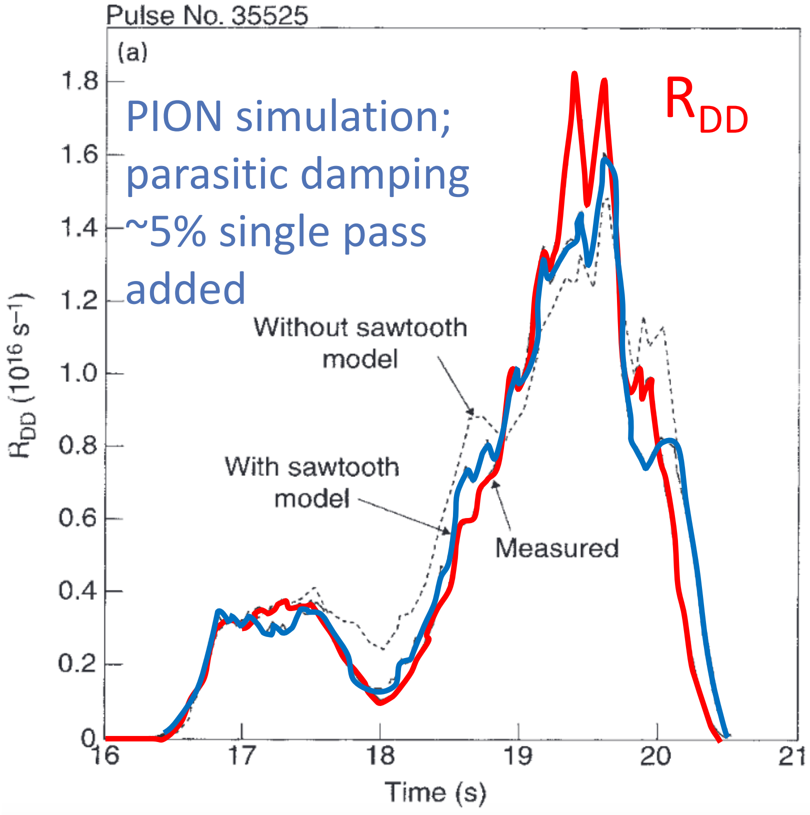
*L. Villard et al., Computer Physics Reports **4**, 95 (1986).

JET Experiment aimed at Fast Wave Current Drive in D plasma $\omega = 3\omega_{cD}$ near centre; Full wave code → very weak thermal D damping.



Tail on D distribution → damping increased due to FLR effects → stronger tail and so on.



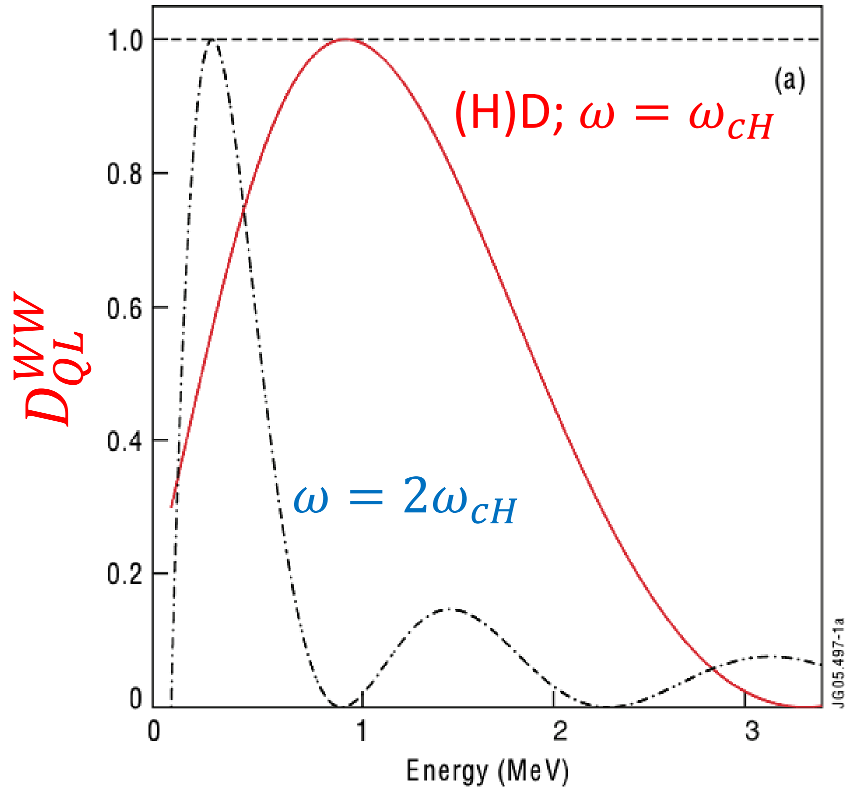


Wave propagation and F-P must be coupled self-consistently.

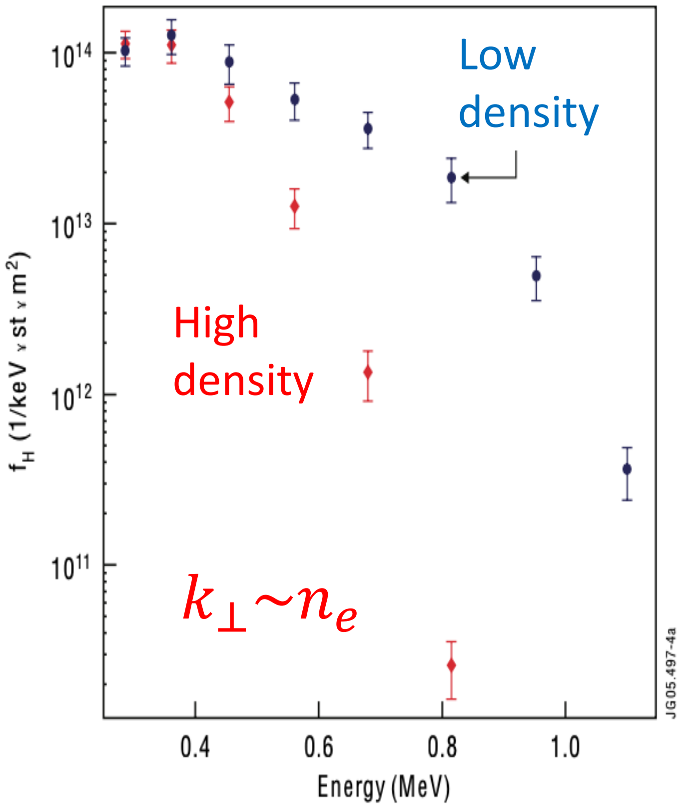
L.-G. Eriksson et al 1998 Nucl. Fusion 38 265

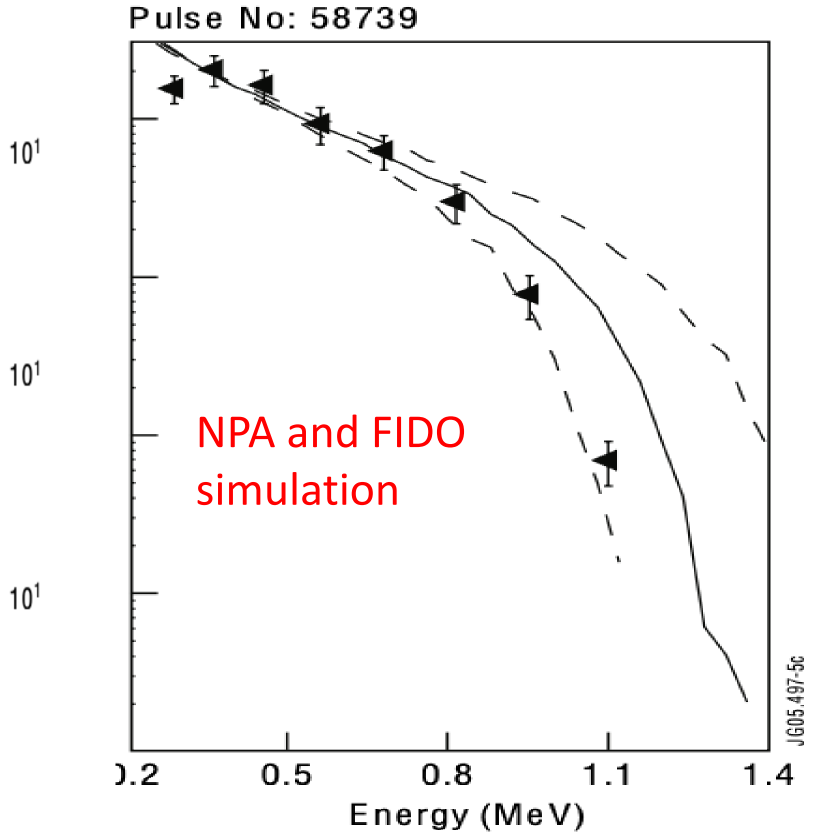
FLR effect; supressed velocity space diffusion

- Second harmonic H heating in JET; $P_{ICRF} \approx 4$ MW



NPA measurement of $f_H(E)$

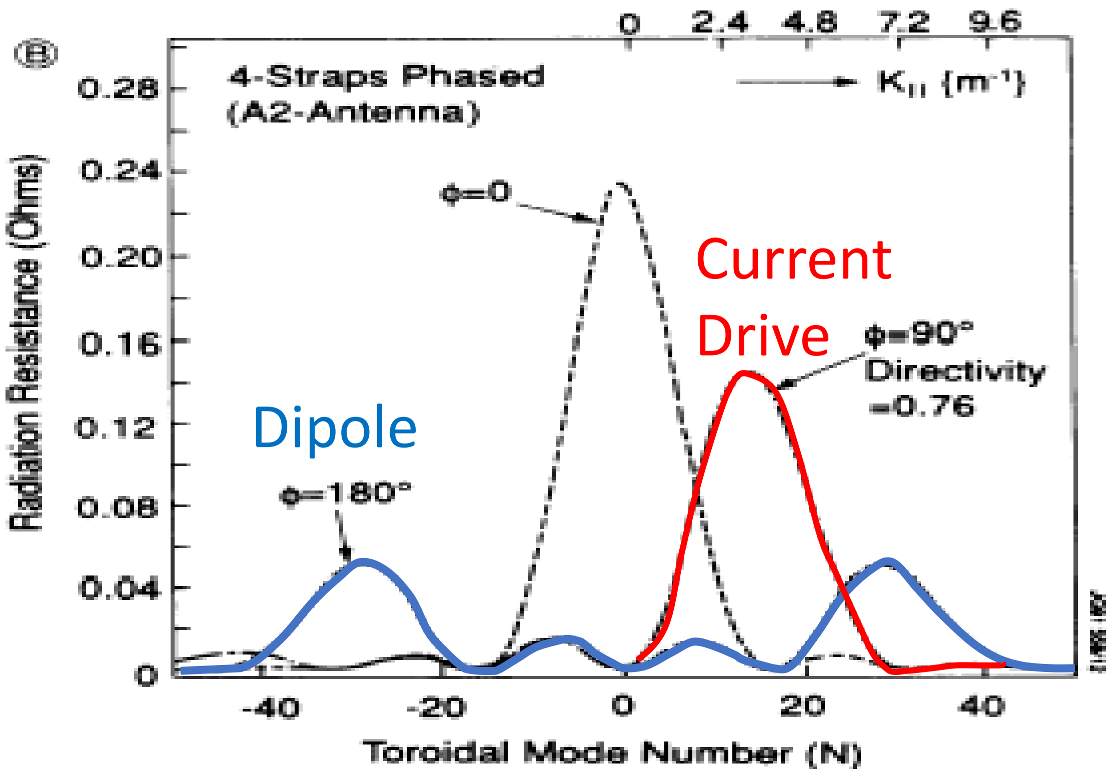




A. Salmi et al., Plasma Phys. Control. Fusion **48** (2006) 717

Influence of asymmetric ICRF antenna spectra

JET



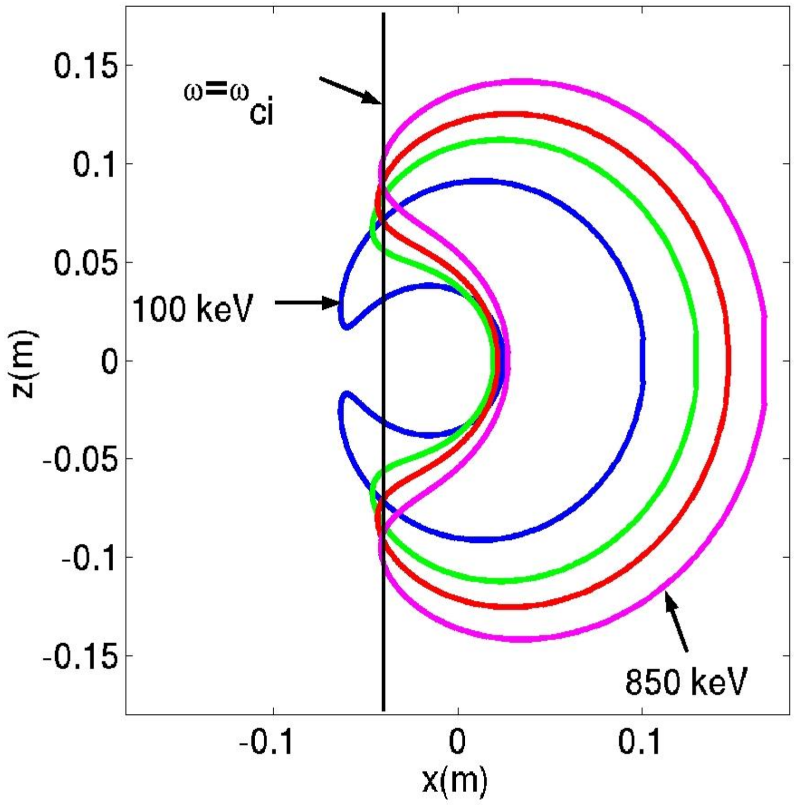
$$\langle N_\varphi \rangle = \frac{\sum_{N_\varphi} N_\varphi P(N_\varphi)}{\sum_{N_\varphi} P(N_\varphi)}$$

$$\frac{dP_\varphi}{dt} = \frac{\langle N_\varphi \rangle}{\omega} \frac{dW}{dt}$$

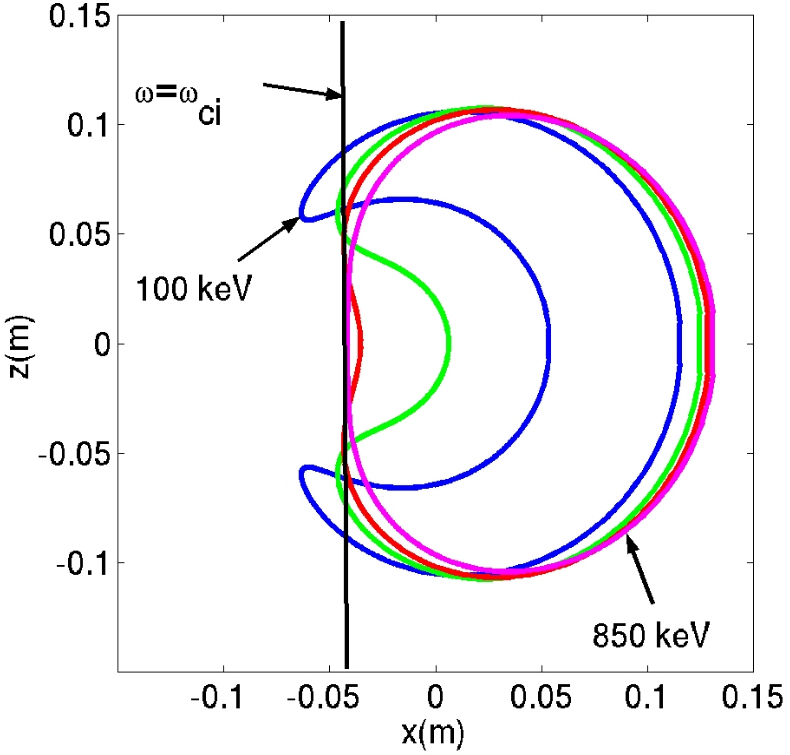
JET +90° phasing → $\langle N_\phi \rangle$~12; co-current propagating waves

Simulation of ^{3}He accelerated from 100 to 850 keV

N_ϕ=-15



N_ϕ=+15

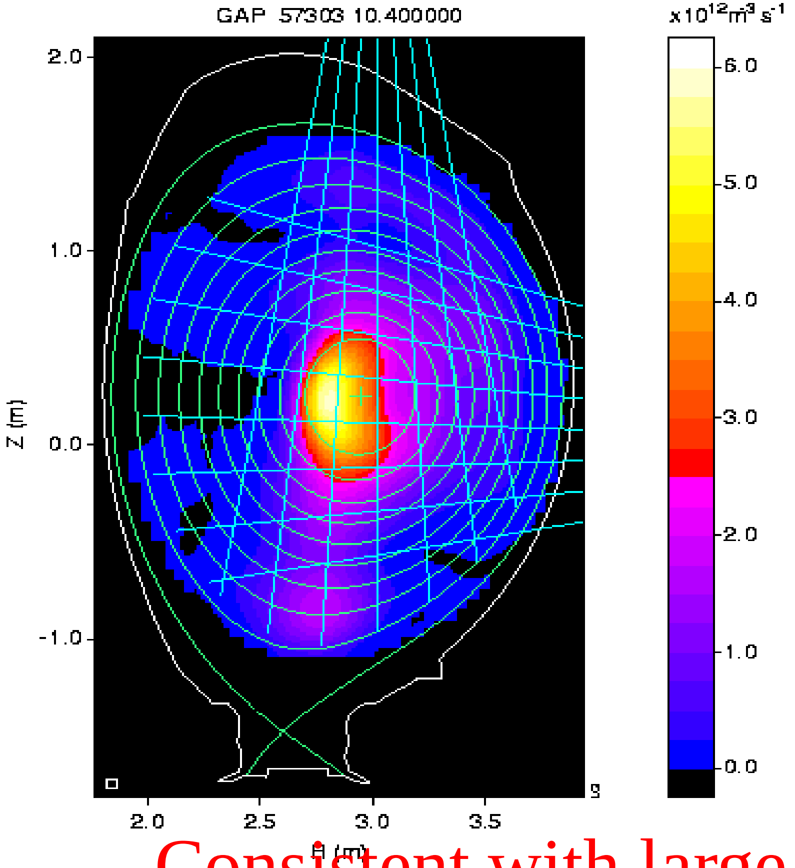


Outward movement of banana tips

Eventual de-trapping into co-passing orbit in the potato regime.

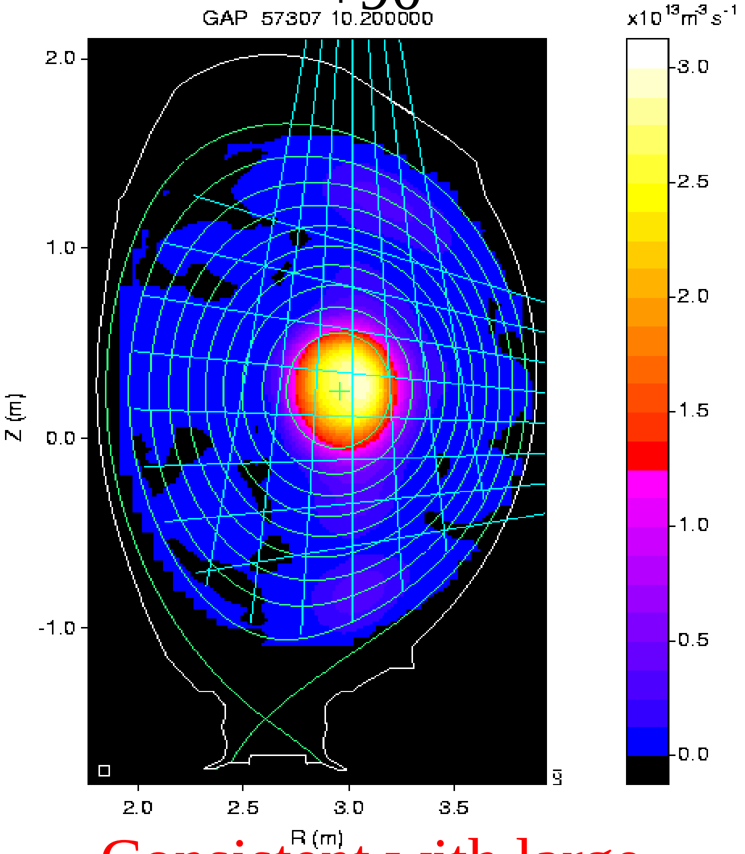
Gamma-ray measurements, interactions between fast ^{3}He ions and C and Be impurities with W(^{3}He) ≳ 1.3 MeV

-90^o



Consistent with large trapped ion fraction

+90^o



Consistent with large fraction of co-passing ions

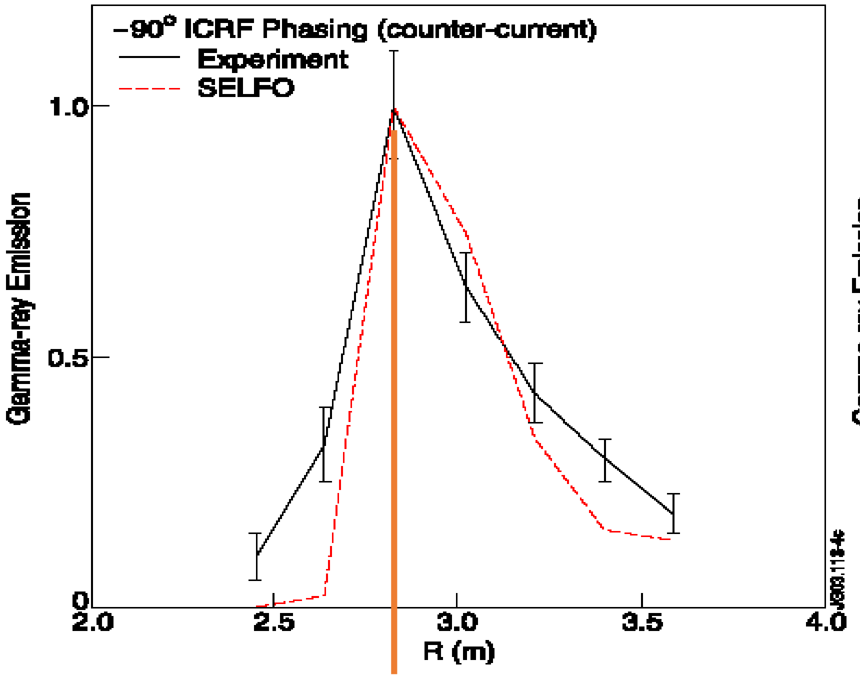
(^{3}He)D heating

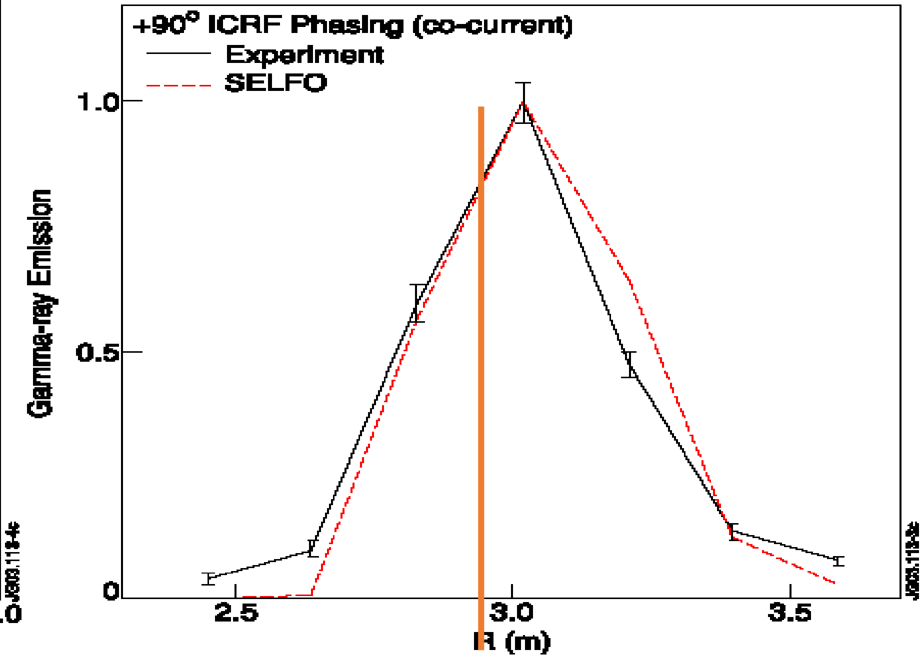
$P_{RF} \approx 5$MW
I_p=2.8MA
B_T=3.4T
f_{ICRF}=37MHz

⇒ Cyclotron resonance slightly on high field side.

M. Mantsinen et al, Phys Rev Lett, 2002, **89,** 115004
L.-G. Eriksson et al, Phys Rev Lett 2004, **92**, 23500

FIDO (ICRF Monte Carlo) code simulation of the gamma-ray emission for vertical lines of sights as a function of the major radius and comparison with measurements





Simulation shows: trapped ions spend much time near their turning points ⇒ asymmetric emission around R=3m.

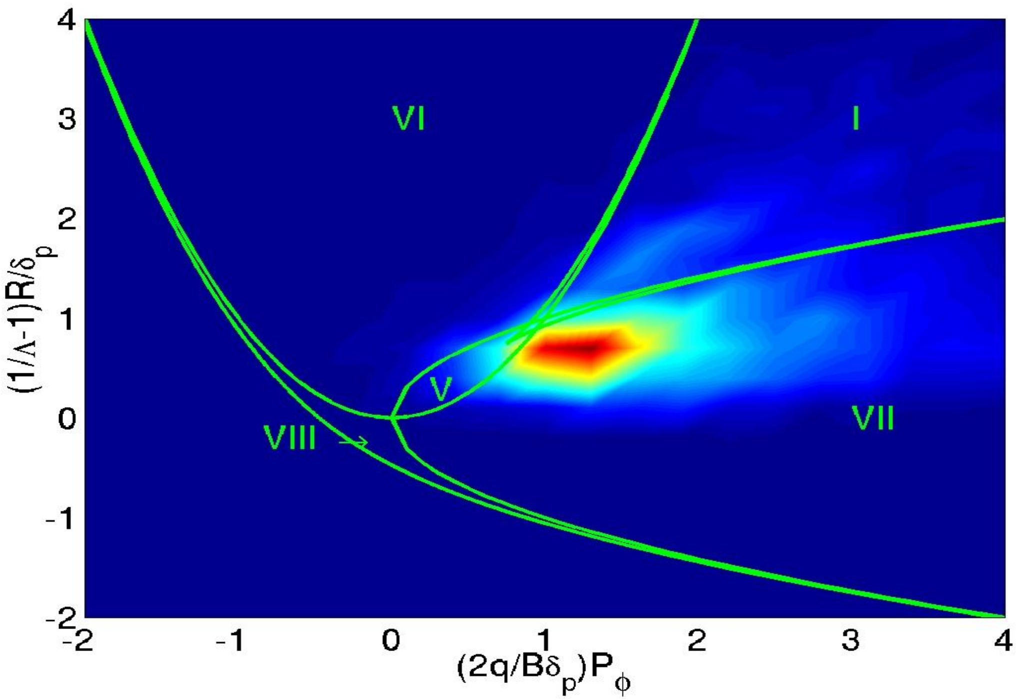
Simulation shows: the more symmetric emission around R=3m is due to ions on co-passing orbits in the potato regime

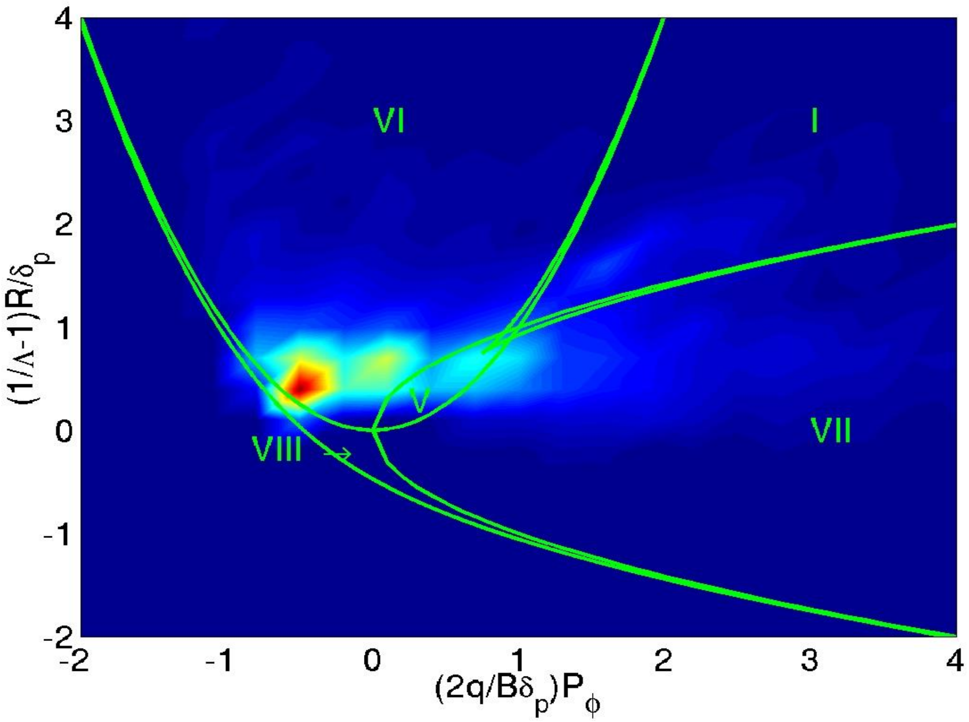
L.-G. Eriksson et al, Phys Rev Lett 2004, **92**, 23500

Simulation with Monte Carlo code FIDO solving orbit averaged Fokker Planck equation; MC particles in an orbit classification diagram; E > 500 keV

-90º

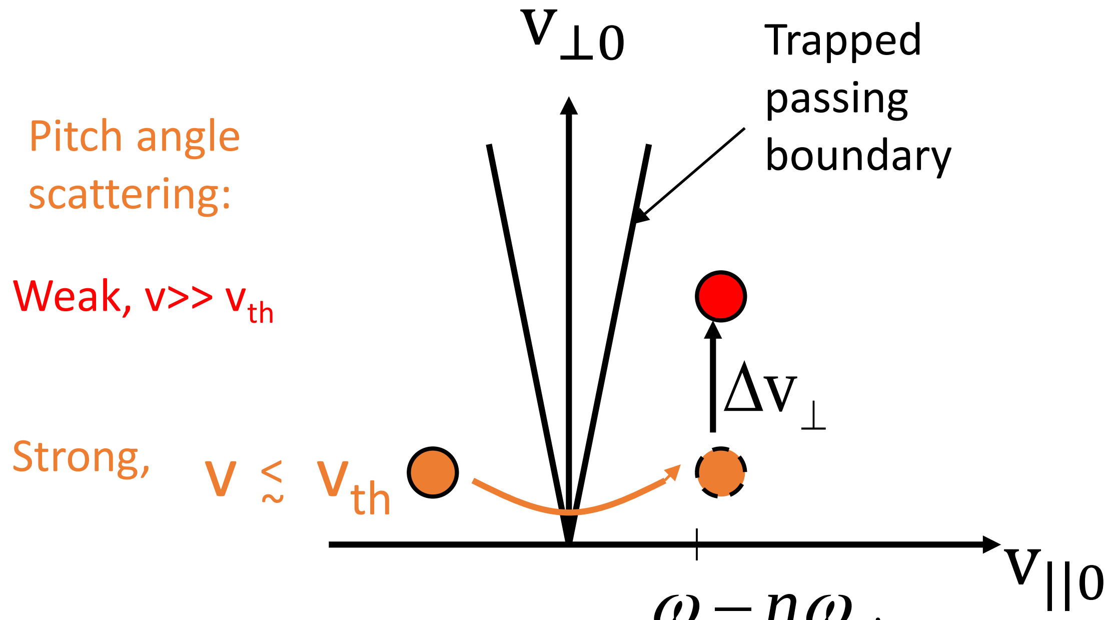
+90º





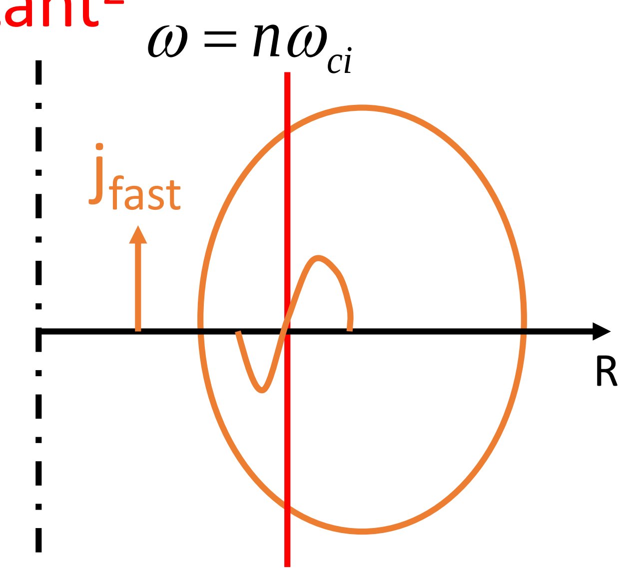
Physics of Ion Cyclotron Current Drive (ICCD)

- Fisch mechanism for ICCD using an asymmetric N_φ spectrum SBW limit
- As already seen, significant modification if FOW is important[2]

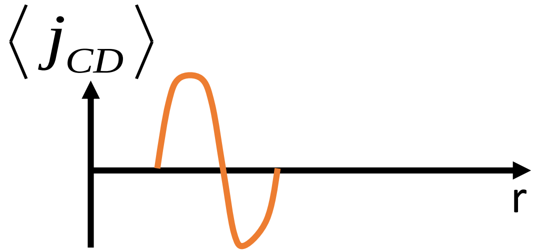


$$v_{||} = \frac{\omega - n\omega_{ci}}{k_{||}}$$

- For $(\omega - n\omega_{ci}) / k_{||} > 0$ (<0) there will be an excess particles with $v_{||} > 0$ $(v_{||} < 0)$, i.e. a driven current.
- The effect is diminished by fast particles entering the trapped region.



$$j_{CD} = j_{fast}\left[1 - \frac{Z_{\min}}{Z_{eff}} + G(\varepsilon)\right]$$



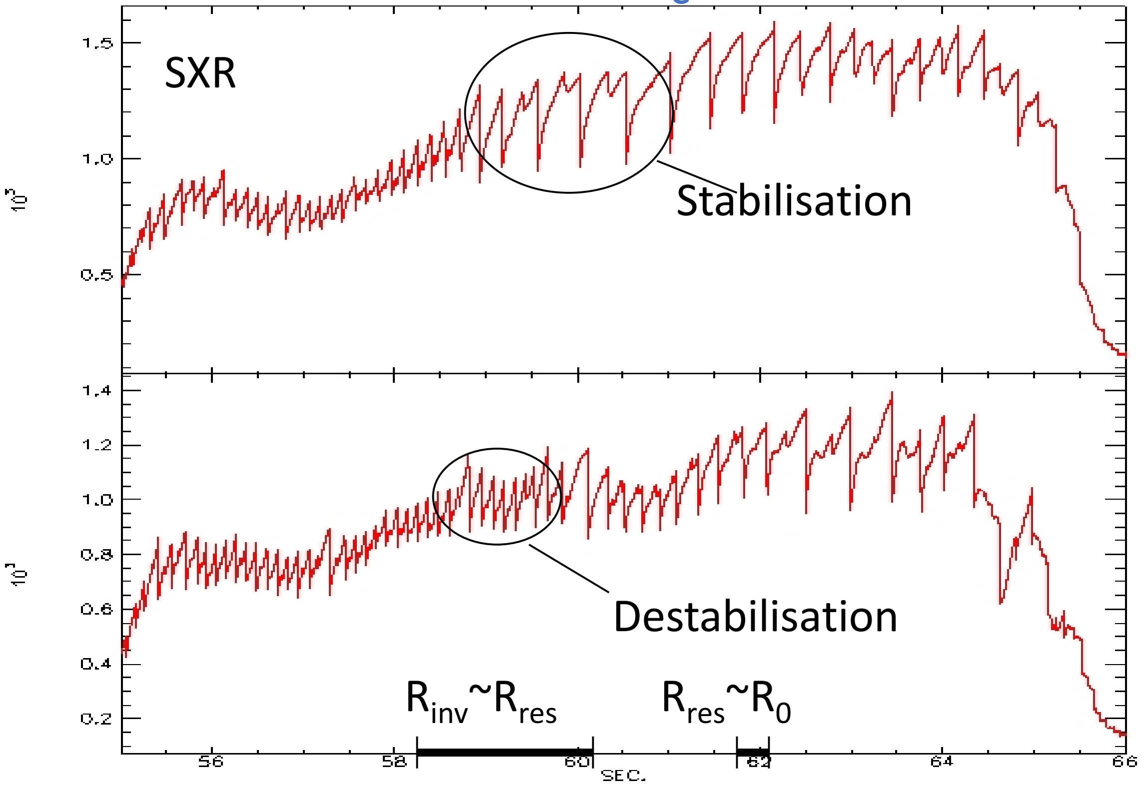
[1]N.J. Fisch, Rev. Mod. Phys. **59**, 175 (1987); [2]T. Hellsten et al., Phys Rev Lett **74**, 3612 (1995).

Sawtooth behaviour with different antenna phasings on JET

- The ICCD current's dipole character should alter the shear $s = \frac{r}{q}\frac{dq}{dr}$
- Altering *s* near the inversion radius should affect the sawtooth frequency[1]
- Orbit topology effects may also play a role[2].

JET H minority ICCD; $R(\omega=\omega_c)$ ramped from HFS to LFS[3]

n_H/n_D~30%



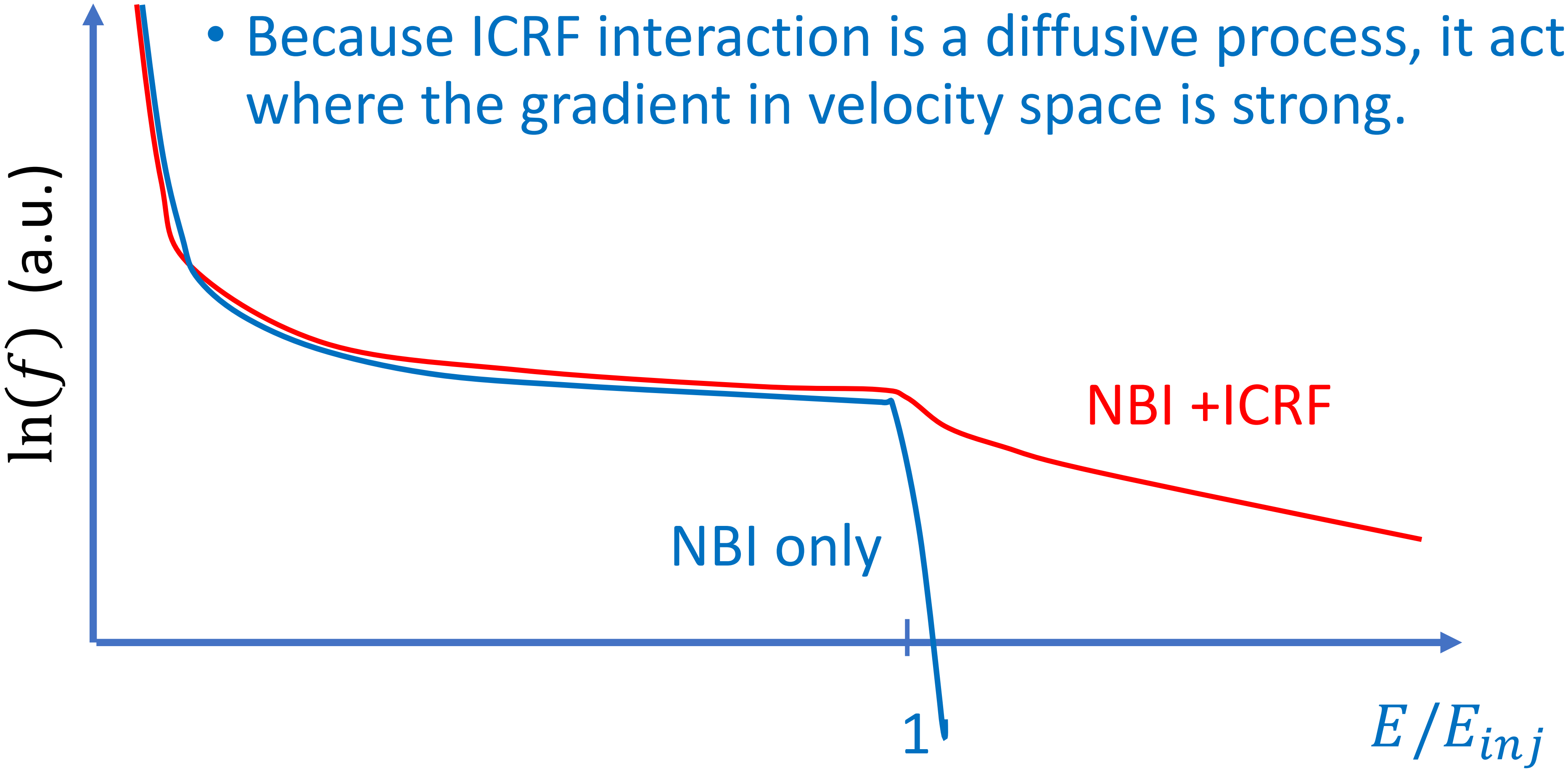
+90°, co-current propagating waves

-90°, counter-current propagating waves

[1]F. Porcelli et al., PPCF **38**, 2163 (1996);
[2]J.P Graves Phys Rev lett, 2009, **102**, 065005
[3]L.-G. Eriksson et al. NF **46,** 951 (2006).

Combined NBI and ICRF heating cartoon

- Because ICRF interaction is a diffusive process, it acts chiefly where the gradient in velocity space is strong.



Non linear phase memory loss and random walk

CHALMERS

- The non linear phase change between two passages of a resonance:

Single N_φ mode: Chirikov criterion:

$$\Delta\phi = \int_{t_{res}}^{t_{res}+\tau_b} \left(\omega - k_{||}v_{||} - n\omega_{c,i}\right) d\tau_b\,, \qquad \gamma = \frac{\partial \Delta\phi}{\partial v_\perp}\Delta W \gg 1$$

- If fulfilled, ϕ is randomised → Random walk process
- Small banana width limit passing particles for n = 1; $k_{||} = 0$[1], $\gamma \approx \dfrac{\omega q r^2 \Delta W}{2mRv_{||}^3}$
- γ decreases with W → Super adiabatic movement at some point.
- Trapped particles are easier to randomise; FOW are important[2].
- A full N_φ spectrum tend to be necessary to explain observed ICRF acc. Ions[2,3]

[1]T.H. Stix "Waves in Plasmas" AIP 1992

[2]P. Helander and M. Lisak Phys. Fluids **B 4** (7), July 1992, 1927

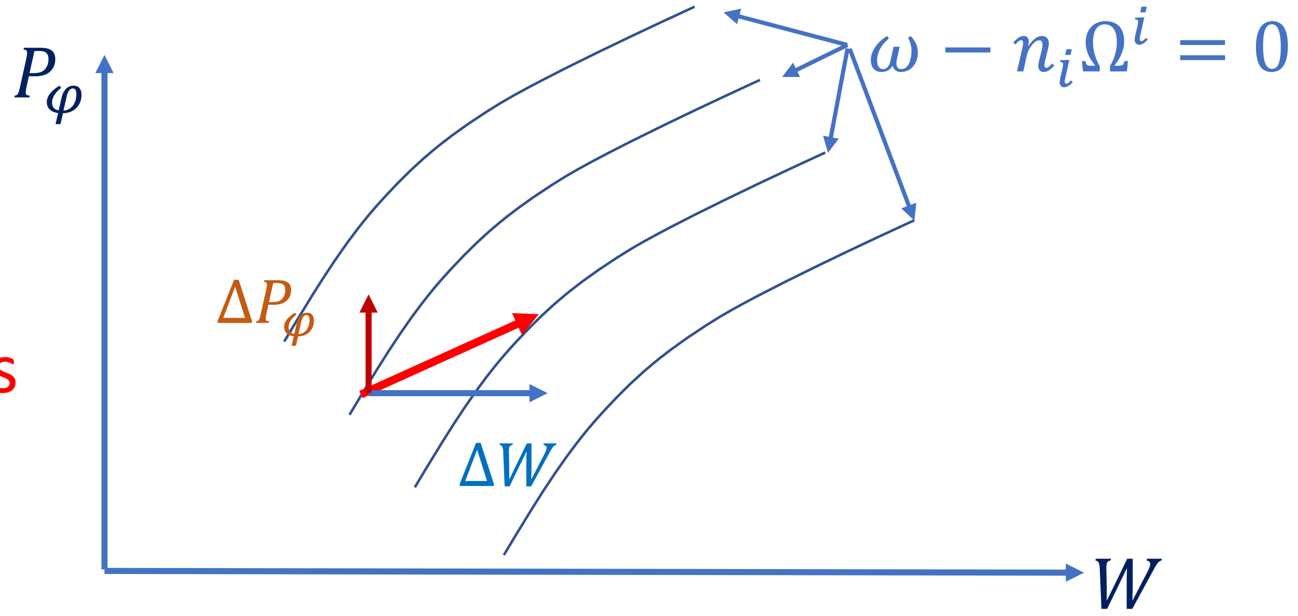
[3]V. Bergaud et al. Phys. Plasmas, **8**, 2001, 139

- A stricter derivation of $\langle Q(f_0)\rangle$ using quasi-linear theory[1,2] yields,

$$D_{QL}^{ww}(n_1 = n, n_3 = N) = \omega_b \sum_{n_2} \frac{\left.\langle(\Delta W)^2\rangle\right|_\phi}{2\tau_b}\delta(\omega - n_i\Omega^i)$$

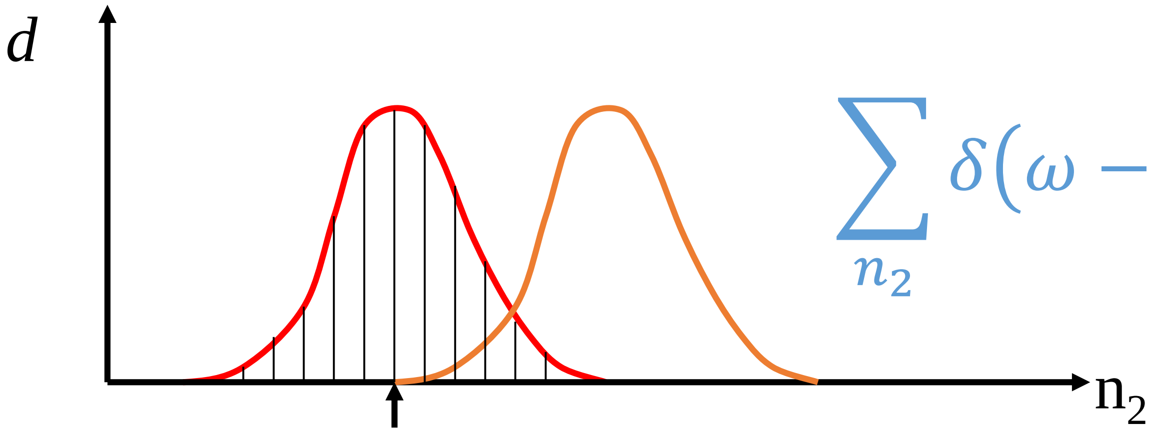
$\Omega^1 = \langle\omega_c\rangle$; $\Omega^2 = 2\pi/\tau_b$; $\Omega^3 = \langle\dot{\varphi}\rangle$; $\langle\cdots\rangle =$ Orbit average

- the resonance condition $\omega - n_i\Omega^i = 0$ picks out particles that "feel" the same wave phase after one unperturbed poloidal orbit (not to be confused with $\omega - \vec{k}\cdot\vec{v}_g = n\omega_{ci}$)
- This global resonance condition, maps out surfaces in $\vec{I}$ space.
- Strong perturbation → kicks bridges resonant surfaces → random walk



[1]A.N. Kaufman Phys. Fluids. 1972;
[2]L-G. Eriksson and P. Helander, Physics of Plasmas **1**, 308, 1994

- In reality the delta functions are broadened by collisions and non-linear effects, as discussed, i.e. $\delta(\omega - n_i\Omega^i) \rightarrow d(\omega - n_i\Omega^i)$
- If the overlap is sufficient one can make the substitution * :



$$\sum_{n_2} \delta(\omega - n_i\Omega^i) \longrightarrow \sum_{n_2} d(\omega - n_i\Omega^i) \approx \frac{1}{\Omega^2} = \frac{1}{\omega_b}$$

$$\omega - n^1\Omega_1 - n^2_{res}\Omega_2 - n^3\Omega_3 = 0$$

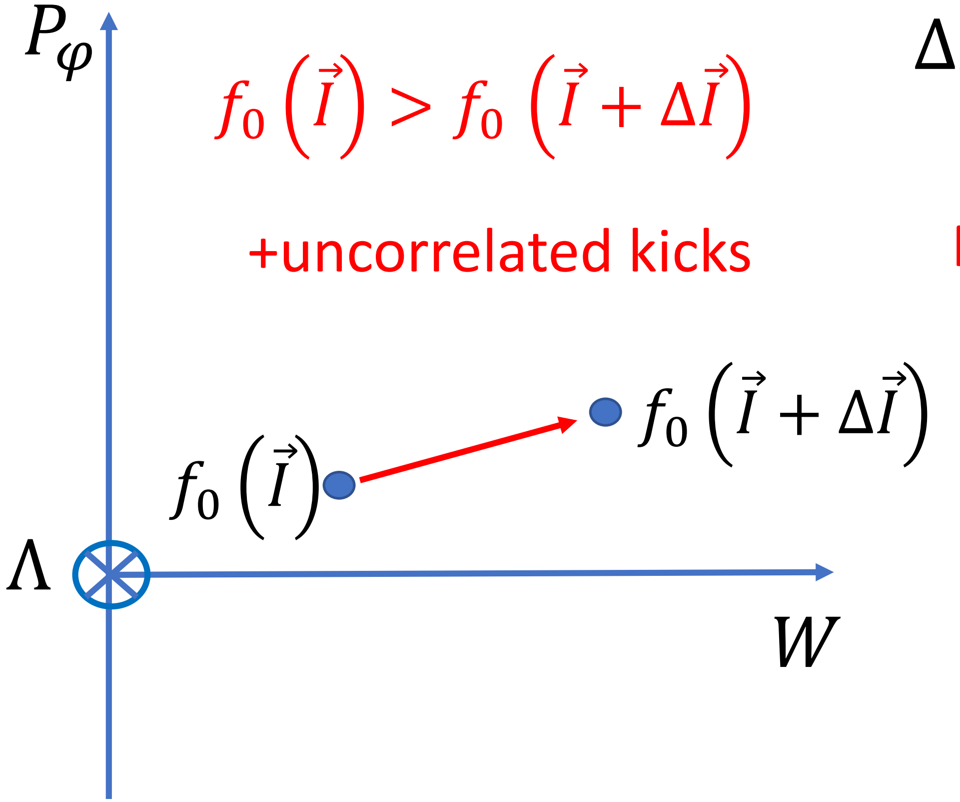
- Thus, if we have "sufficient" overlap, i.e., $\gamma \gg 1$ we recover :

$$D_{QL}^{WW} = \sum_{res} \frac{\langle(\Delta W)^2\rangle\big|_{\phi}}{2\tau_b}$$

*Becoulet at al. Phys. of Fluids **B3** (1991),

Cartoon heating vs instability

Heating



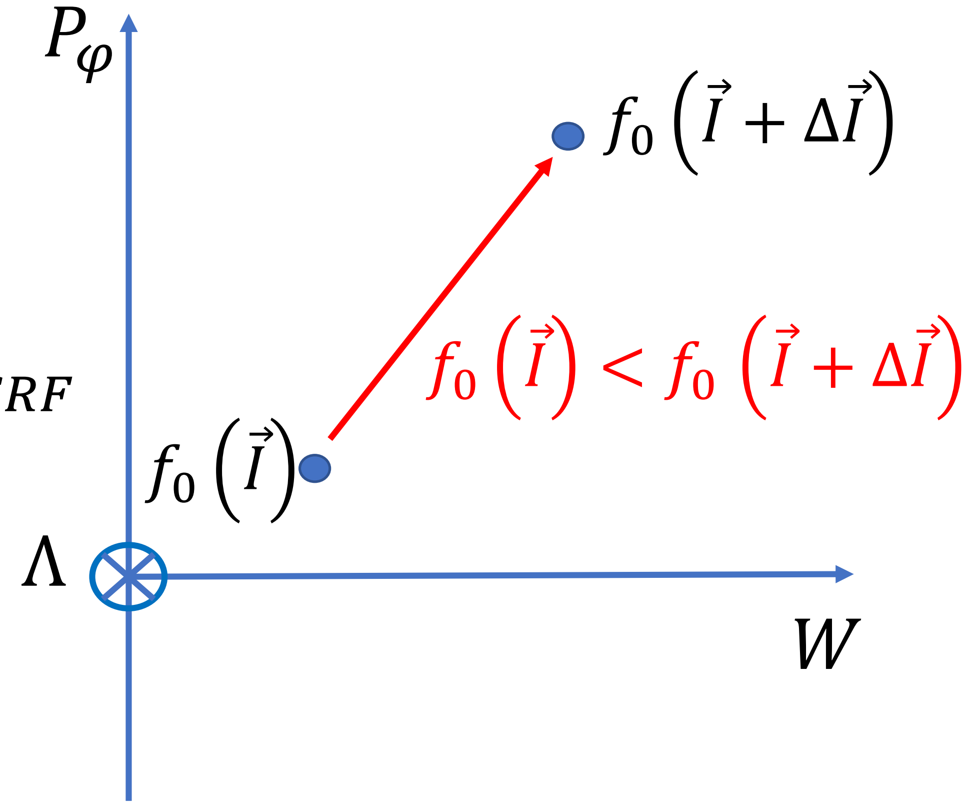
$$\Delta\vec{I} = (\Delta W, \Delta\Lambda, \Delta P_\varphi)$$

$$\Delta P_\varphi = \frac{N}{\omega}\Delta W$$

Note:

$$\omega_{TAE} \ll \omega_{ICRF}$$

Potential for instability



Round off

- Fast ion physics is an area with many facets
- In this lecture it was only possible to touch on selected subjects of fast ions sources somewhat superficially
- However, questions like to which extent a different bulk plasma species are heated by energetic ions, the currents they fast ions can drive etc. is useful general knowledge when e.g. assessing plasma scenarios.

- Finally, energetic ions are important but so are energetic electrons, not least runaway electrons and I encourage you to follow the lectures on Friday attentively.

Thank you for your attention!

Is the kick only in the perp. Direction?

- The change in the parallel energy is given by,

In general $\neq 0$

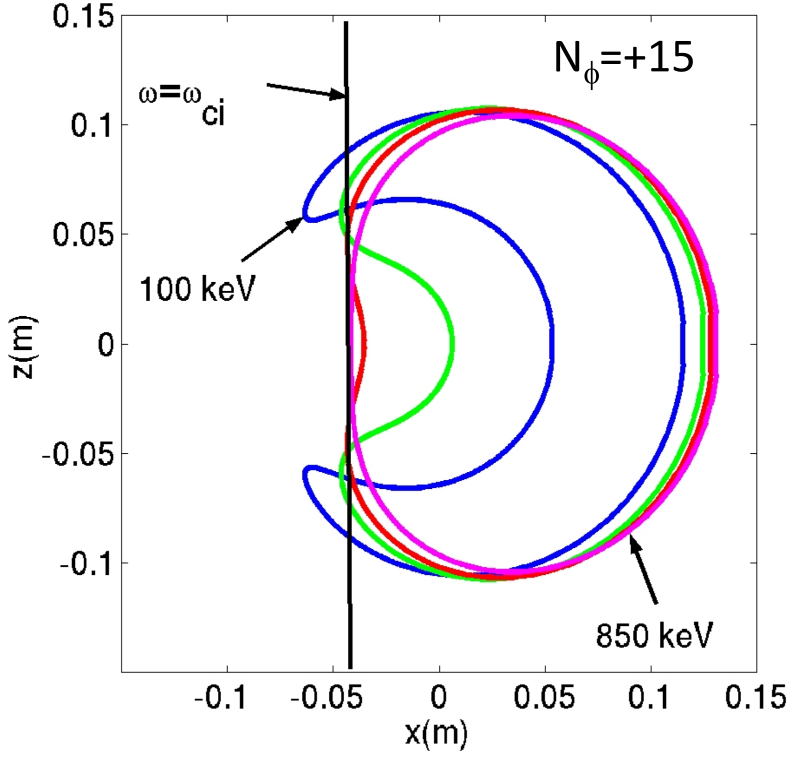
$$\Delta \frac{m v_{||}^2}{2} = \int_{t_0-\Delta t}^{t_0+\Delta Z} Ze\left(\vec{E} + \vec{v} \times \vec{B}_1\right) \cdot \vec{v}_{||} dt$$

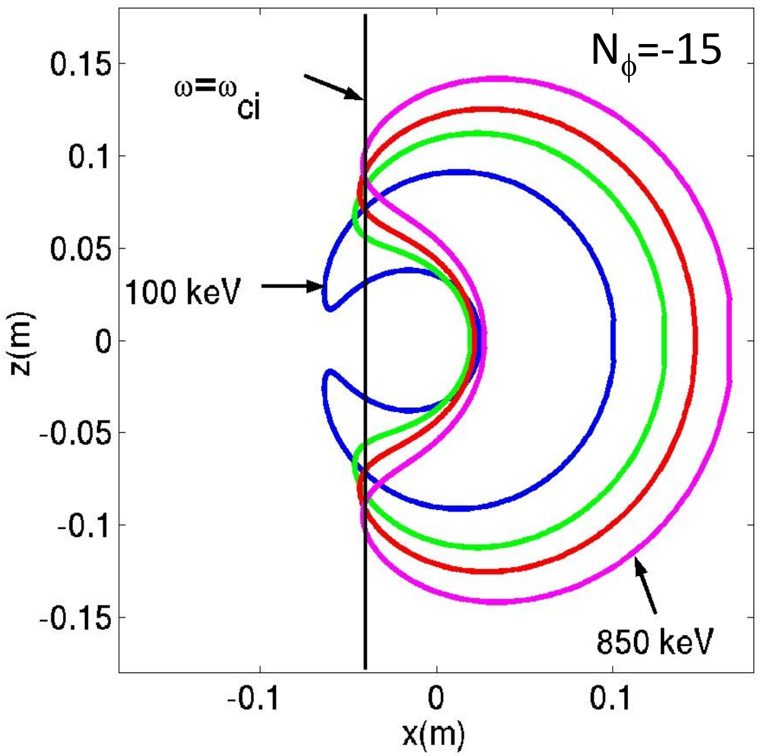
$$= \int_{t_0-\Delta t}^{t_0+\Delta t} Ze\left[E_{||} v_{||} + \vec{B}_1 \cdot \left(\vec{v}_{||} \times \vec{v}_{\perp}\right)\right] dt$$

- Thus, while the $\vec{v} \times \vec{B}_1$ term obviously is not involved in the total energy kick, it can redistribute part of it to the parallel direction.
- We can save a lot of tedious calculations by adopting a quantum mechanical perspective.

The Physics ICCD is more complicated

H ion accelerated from 100 to 850 keV





Eventual de-trapping into co-passing orbit in the potato regime ⇒ passing current not described by "Fisch like" theory*. Increased pressure in the centre.

Outward movement of turning points, decrease of central fast ion pressure.

- Finite orbit width effect of trapped ions → dipole effect on driven current (co-current further out)
- Change of orbit topology, i.e. ions driven into non-standard passing orbits

*T. Hellsten, et al. Phys. Rev. Lett. **74**, 3612 (1995).

Physical intuition of P_φ

- The equation of motion in the toroidal direction reads,

$$m\frac{d(Rv_\varphi)}{dt} = Ze\left(\vec{v}\times\vec{B}\right)_\varphi = -ZeRv_rB_\theta$$

- The poloidal flux function is given by,

$$\psi = \int_0^r RB_\theta\, dr' \;\Rightarrow\; \frac{d\psi}{dt} = \vec{v}_r\cdot\nabla\psi = RB_\theta v_r \;\Rightarrow$$

$$\boxed{\frac{d}{dt}\left(mRv_\varphi + Ze\psi\right) = \frac{dP_\varphi}{dt} = 0}$$

- One can see $Ze\psi$ as a "potential angular momentum"
- For trapped particles one should note that: $P_\varphi = Ze\psi_{t.p.}$ where "t.p." stands for turning point.

Small Larmor radius

- Define orbit average:

$$\langle \cdots \rangle = (2\pi)^{-3} \iiint_0^{2\pi} (\cdots)\, d\theta^1 d\theta^2 d\theta^3 \approx \frac{1}{2\pi}\int_0^{2\pi} (\cdots) d\theta^2 = \frac{1}{\tau_b}\int_0^{\tau_b} (\cdots) d\tau$$

- and the orbit average of the Fokker-Planck equation yields

$$\frac{\partial f_0}{\partial t} = \langle C(f_0) \rangle + \langle S \rangle - \langle L \rangle + \langle Q(f_0) \rangle$$

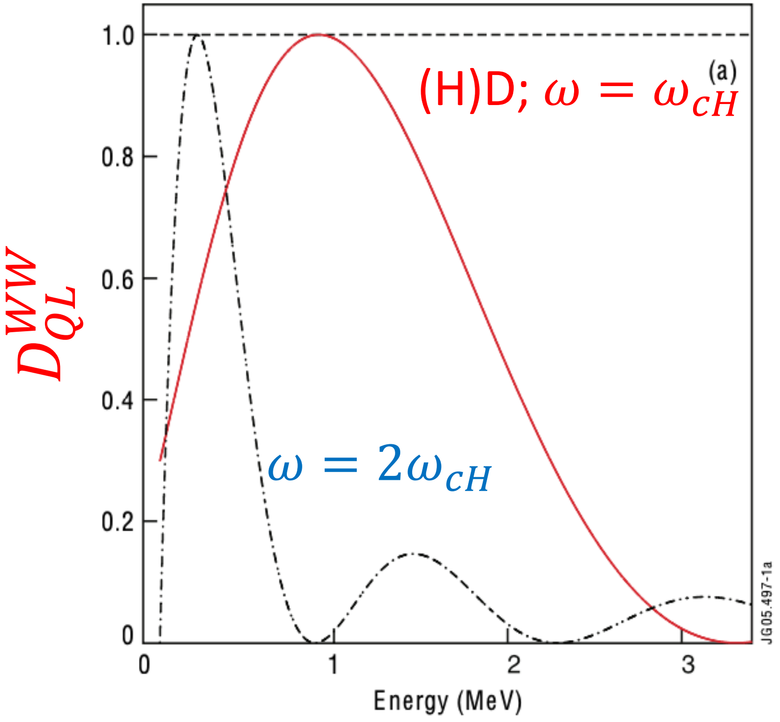
- Note $\langle f \rangle = f_0\left(\vec{I}\right)$, with $\vec{I} = \vec{I}(\vec{J})$
- The collision operator is conservative → $C(f) = \frac{\partial \overline{\Gamma}_c^i}{\partial z^i}$
- With use our favourite variables, $\vec{I} = \left(v, \Lambda = \frac{\mu B_0}{E}, P_\varphi\right)$

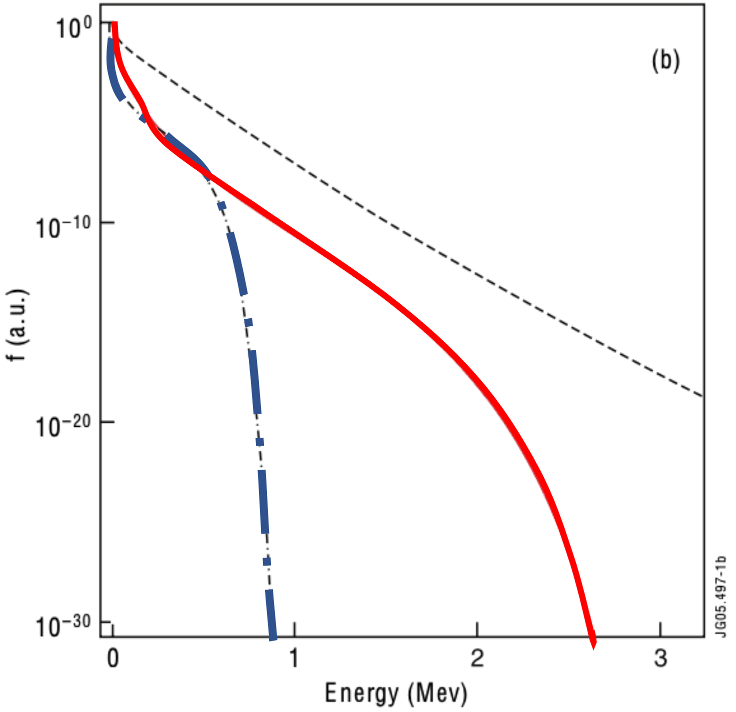
$$\langle C(f) \rangle = \frac{1}{\sqrt{g}} \frac{\partial}{\partial I^i}\left[\sqrt{g}\left\langle \overline{\Gamma}_c^j \frac{\partial I^i}{\partial z^j} \right\rangle\right]$$

$$\sqrt{g} = \left|\frac{\partial \vec{z}}{\partial(\vec{I}, \vec{\theta})}\right| = \frac{v^3 \tau_b}{4\pi m \omega_{c0}}$$

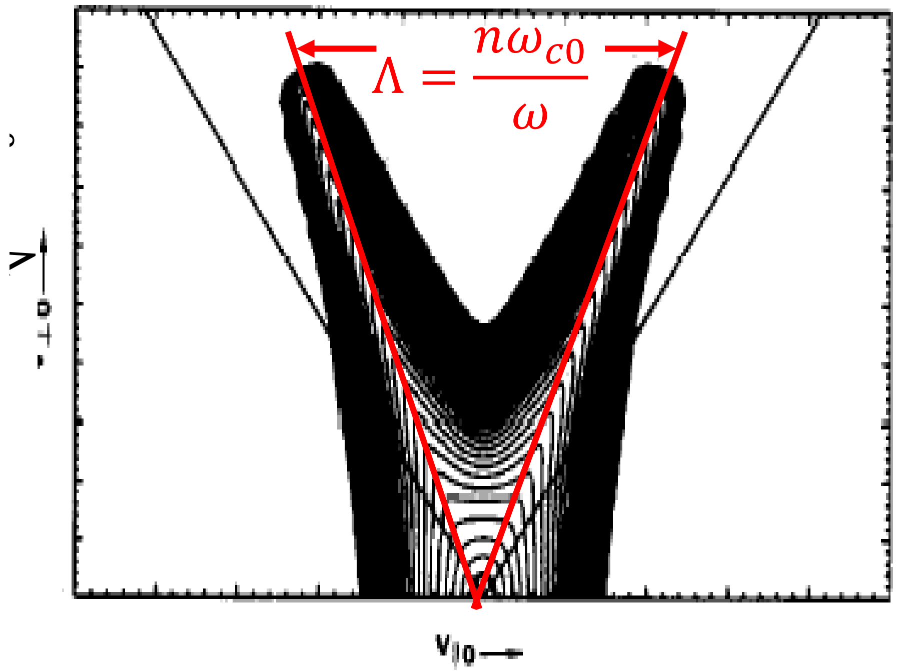
Properties of ICRF heated distributions

- The collisions are much stronger at low energies than at high → development of a non-Maxwellian tail on the distribution.
- $\Delta\Lambda = \frac{n\omega_{c0} - \Lambda\omega}{\omega W}\Delta W \rightarrow \Lambda \xrightarrow[\Delta W>0]{} \frac{n\omega_{c0}}{\omega}$; ion with t.p. at resonance, has $\Lambda = \frac{n\omega_{c0}}{\omega}$





Small Banana width limit



G.D. Kerbel and M.G McCoy, Phys. Fluids **28** (1985) 3629.